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Vanguard Asset Allocation Model*: An investment solution for active-passive-factor portfolios

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- Mean-variance optimization and other conventional portfolio construction approaches operate in two dimensions: portfolio risk and portfolio return. However, real-world investor decisions suggest that portfolio selection depends on the intersection of multiple dimensions of risk and return, from systematic risk and volatility to active alpha, tracking error, and implicit risk factor exposures.

- The Vanguard Asset Allocation Model (VAAM), a proprietary model for determining asset allocation among active, passive, and factor investment vehicles, simultaneously optimizes across the three dimensions of risk-return trade-offs (alpha, systematic, and factor). The model incorporates Vanguard's forward-looking capital market return and client expectations for alpha risk and return to create portfolios consistent with the full set of investor preferences.

- The model can solve portfolio construction problems conventionally addressed in an ad hoc and suboptimal manner. It yields more appropriate answers to common investor objectives and asset allocation problems. These answers include: (1) strategic multiasset model portfolios, such as passive-only, passive-factor, and passive-factor-active portfolios; (2) tailored strategic multiasset portfolios that reflect an investor's risk tolerance, investment horizon, and other investment constraints; (3) time-varying active-passive-factor portfolios, with allocation changes driven by specific economic scenarios and market conditions; and (4) active manager substitution analysis, solving for lower-cost passive and factor portfolios as a substitute for high-cost active portfolios.

* Patent pending.

Asset allocation and the need for an active-passive model

The active-versus-passive management debate has been explored extensively in the investment literature. The "zero-sum game" and the underperformance of the average active manager net of costs are clear (see for example Sharpe, 1991, and Rowley et al., 2017). Even so, many real-world investors still allocate at least some portion of their portfolios to active managers.

This behavior is not necessarily a sign of poor decision-making; rather, its prevalence reveals that conventional portfolio construction approaches might fail to account for the full range of investor preferences and beliefs. After all, the idea of a zero-sum game implies that half of the active managers must outperform the benchmark before costs. Thus, investors who use active funds in their portfolios must believe, with some degree of conviction, that they can select managers from the "right half" of the distribution.

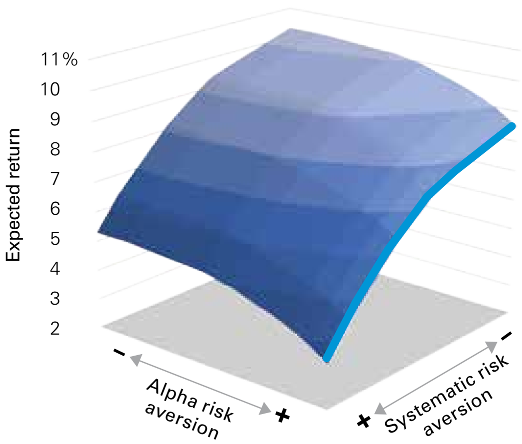
In 2017, Vanguard introduced a framework to help investors decide how to allocate across active and passive investments in their portfolios (see Wallick et al., 2017). The Vanguard active-passive framework moves past the traditional active-passive debate; instead, it lays out the conditions under which it makes sense for investors to bring both active and passive investments together in a portfolio.

A critical element of the framework is that it explicitly considers alpha risk and an investor's attitude toward it in the construction of active-passive portfolios. Traditional quantitative approaches, such as the mean-variance optimization (MVO) pioneered by Markowitz (1952), often suffice for solving passive, long-only portfolio problems, but they face limitations once active or factors are added to the menu of choices. Most important among these limitations is that alpha risk and associated risk aversion are ignored.[1] Extending the traditional MVO efficient frontier into this missing dimension of alpha risk aversion would generate a three-dimensional efficient frontier—an efficient surface—as illustrated in **Figure 1a**. The traditional MVO efficient frontier, shown in **Figure 1b**, can be thought of as a particular segment of the efficient surface, one where an investor is extremely averse toward alpha risk.

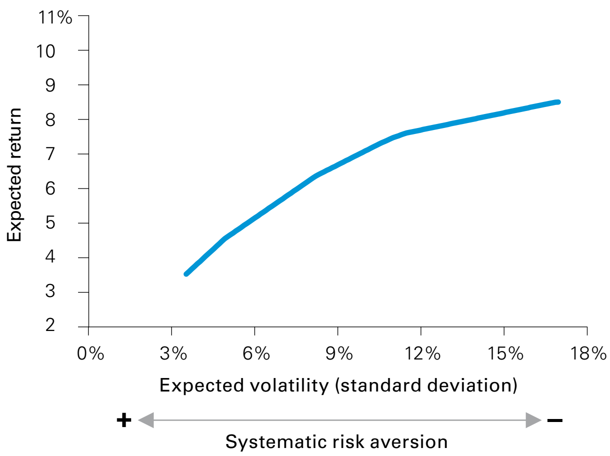
Other, more ad hoc approaches use a sequential decision-tree structure to try to handle the active and/or factor dimensions of a portfolio. These methods usually break the problem into three steps: (1) determine the passive

Figure 1. The missing link: Alpha risk aversion

a. Efficient frontier with alpha risk



b. Traditional MVO efficient frontier



Note: For illustrative purposes only.
Source: Vanguard.

1 In this paper we define *alpha* as the idiosyncratic return component that cannot be explained by either market exposure or factor risk exposures. Alpha is entirely explained by an active manager's security selection and/or market timing skill.

allocation among broad asset classes; (2) determine the allocation of sub-asset classes and factor tilts within each broad asset class; and (3) determine the active-passive split around each (passive) benchmark. However, such an approach can't address the investment trade-offs that investors confront across the layers of alpha, systematic, and factor risks. Nor can it accommodate varying levels of risk aversion across investors. In addition, relative to other quantitative portfolio frameworks, it is vulnerable to inefficient use of information, including asset-return expectations, volatilities, correlations, factor loadings, or tracking errors.

The Vanguard Asset Allocation Model (VAAM), which grew out of the need to help investors meet such portfolio construction challenges, determines the optimal allocation across active, passive, and factors in a portfolio. It is an expected utility-based model that assesses risk and return trade-offs of various portfolio combinations based on user-provided inputs such as risk preference, investment horizon, and which asset classes and active strategies are to be considered. The VAAM is integrated with the Vanguard Capital Markets Model® (VCMM), as it takes as inputs VCMM-generated forward-looking return expectations at various horizons. In addition to the optimal portfolio, the VAAM generates a range of portfolio metrics, including forward-looking risk and return distributions of the portfolio, expected maximum drawdown, and the probability of returns being above a given level.

Although it draws on the logic of the Vanguard active-passive framework, the VAAM is a full-fledged investment solution that can be applied to solving real-world portfolio problems. It can answer many investor questions, such as: How do I simultaneously determine active-passive combinations across the multiple asset classes in my portfolio? How does the active-passive decision in one asset class affect the portfolio's overall asset allocation? If I start with, for example, a 60/40 stock/bond passive portfolio, should I include a small allocation to active in the equity portion—or keep the portfolio passive and just increase the equity allocation? How should I account for active managers' factor styles in the portfolio? To what extent can a portfolio's active strategy be replaced by some combination of passive factor investment vehicles—and what might that combination of factors look like? Finally, how do the answers to each of these questions vary (1) across investors with different investment horizons or different attitudes toward risk, and (2) over time, under changing market and economic conditions (such as a rising rate environment or low-growth environment, or during a period of high inflation)?

In this paper, we first discuss the different sources of traditional active fund returns that are key to the active-passive allocation solved with the VAAM. We then provide an overview of the model, describing key inputs such as asset-return expectations, portfolio constraints, and investor attitude toward various dimensions of risk. The third section brings it all together and illustrates the sensitivity of VAAM-customized portfolios to a full range of potential investor inputs. We then present multiple portfolio applications of the VAAM, such as factor model portfolios, active manager substitution analysis, time-varying asset allocation portfolios, and portfolio recommendations under different economic scenarios. In the fifth and final section, we lay out some caveats, as well as the model's limitations, before offering some conclusions.

The anatomy of an active fund

Three key attributes of any active strategy need to be addressed in the active-passive allocation problem solved by the VAAM:

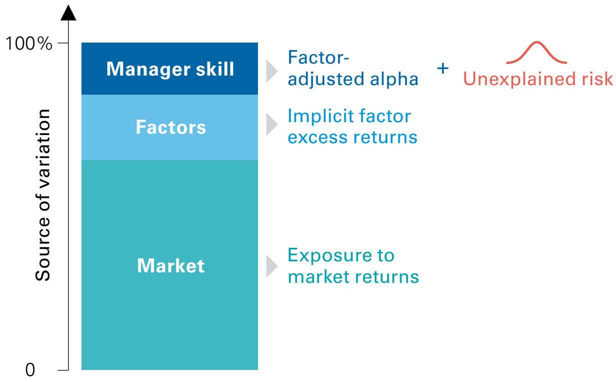
- Factor-adjusted alpha
- Alpha risk
- Investors' alpha risk tolerance

Factor-adjusted alpha: The true measure of manager skill

Should an active fund manager be given credit for outperformance arising from systematic tilts toward factors that attempt to harvest risk premia over long periods? Factor-based investing has been well known for decades, and factor-based products give investors the opportunity to harvest risk premia over long horizons. This also means that active managers' performance can be replicated, at least in part, through factor exposures; Bender, Hammond, and Mok (2014) have shown that up to 80% of the alpha generated by U.S. equity active managers can be explained by exposures to equity risk factors. Similarly, research done by Roberts, Paradise,

IMPORTANT: The projections and other information generated by the VCMM regarding the likelihood of various investment outcomes are hypothetical in nature, do not reflect actual investment results, and are not guarantees of future results. Distribution of return outcomes from VCMM are derived from 10,000 simulations for each modeled asset class. Simulations as of June 30, 2018. Results from the model may vary with each use and over time. For more information, please see "About the Vanguard Capital Markets Model" on page 17.

Figure 2. Factor decomposition of active returns



Note: For illustrative purposes only.
Source: Vanguard.

Figure 3. Factor decomposition of a U.S. equities active fund (Ordinary Least-Squares regression)

Manager skill	
Factor-adjusted annualized alpha (%)	0.81* *(0.000)*
Market beta and factor loadings	
Market beta	0.99** *(0.014)*
Value factor	0.04 *(0.031)*
Mid-cap factor	0.31** *(0.038)*
Return-based regression statistics	
Degrees of freedom	357
Adjusted R-squared (%)	93.51
Tracking error (%)	4.03
Information ratio	0.20

Notes: Standard errors are in parenthesis and refer to monthly frequency data. * indicates a p-value of less than 0.05; ** indicates a p-value of less than 0.01. The active fund shown here was selected from the oldest share class of all available U.S. equities active managers' funds that show a historical factor-adjusted annualized alpha greater than 50 basis points and at least one statistically significant factor loading, using the Russell 1000 Index as the market benchmark. In this paper we focus on U.S. equities style factors only. The value and mid-cap factors have been constructed based on a bottom-up selection of Russell 1000 Index stocks. See Appendix B for further details.

Sources: Vanguard calculations, using monthly data from Morningstar, Inc., from December 31, 1987, through December 31, 2017.

and Tidmore (2018), among others, suggests that the majority of returns for active fixed income managers is explained by exposure to credit and high-yield securities—not by market timing or security selection. Thus, in assessing active manager skill, security selection and timing ability should be taken into account. After all, factor access can usually be gained at lower cost than typical active management fees.

Using a risk-factor attribution least-squares regression (see Sharpe, 1992, Fama and French, 1993, and Chin and Gupta, 2017), **Figure 2** shows how active fund returns can be decomposed into a market component (or systematic risk), a risk factor component, factor-adjusted alpha, and the unexplained return variation (or tracking error). This approach for estimating the factor-adjusted alpha can be a valuable tool for investors in assessing active fund managers and the value they add.[2]

Figure 3 shows the return decomposition for a real-world U.S. equity active fund and uses a month-end return data series that spans the 30 years ended December 31, 2017. The active fund shows a strong factor-adjusted outperformance, with an average factor-adjusted alpha of 81 basis points (bps) per year and a tracking error of roughly 4%. Thus we know the fund manager has added value by security selection and timing, beyond traditional factor and market exposure. In this example, the active fund shows exposure to the mid-cap factor and a slight amount of value tilt.[3]

Alpha risk: The uncertainty around factor-adjusted alpha

Investors willing to invest in active funds must expect some degree of outperformance relative to passive alternatives; implicitly or explicitly, they must have a positive factor-adjusted alpha expectation. However, this alpha expectation is an ex ante estimate or belief, and by no means is it certain to be borne out. Even successful active managers, who generate a positive alpha in excess of their factor-adjusted benchmark on average, often experience periods of underperformance.

2 In this paper, we focus on equity style factors only. For further details on how we defined U.S. equity style factors for the purposes of this paper, see Appendix B. The approach and methodology that we propose can be applied to any definition of factors and across different asset classes, including typical fixed income factors (for example, duration and credit) and factor replication for alternative strategies (for example, hedge funds).

3 Based on the Ordinary Least-Squares (OLS) regression of the historical active manager returns, the value factor loading in this example is not statistically significant (p-value = 21.8%). However, we keep it to highlight how our model would work for investors who are willing to have a value factor exposure in their portfolio, either implicitly through an active manager exposure or explicitly through a passive factor investment.

Figure 4 illustrates the concept of alpha expectation and alpha risk by representing the active manager performance in terms of a probability distribution. As the figure shows, even with a positive alpha expectation (shown as a dotted orange line), it is possible for the active fund to underperform its passive counterpart (the dark blue area to the left of the passive benchmark return).

Investors tend to be risk averse; they dislike this under-performance risk and attempt to trade it off against the positive outcomes (the light blue area of the distribution).

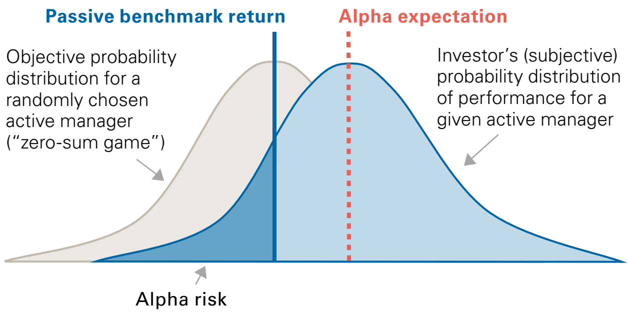
Thus, alpha expectation and alpha risk both have a straightforward statistical interpretation in terms of the standard deviation and mean derived from the bell curve of potential performance outcomes.[4] This distributional interpretation of active manager skill has often been missing in the traditional active-passive debate, where manager's alpha is typically thought of in terms of a point forecast.[5]

Alpha risk aversion: The investor's attitude toward alpha risk

Wallick et al. (2017) discussed the role of this dimension of alpha risk, and investors' associated risk preference, in making active-passive decisions. The degree to which investors dislike the alpha risk—the possible underper-formance in pursuit of outperformance—is their alpha risk aversion. Just as investors display an aversion toward systematic risk (for example, risk aversion to equity compared with cash), they can also display an aversion toward alpha risk.

These two risk aversions can differ drastically from one investor to another. The range of risk budgets (translated as allowable active allocation) seen in the policy portfolios of institutional investors hints at varying levels of alpha risk aversion. The trade-off between expected alpha and alpha risk is also driven by alpha risk aversion. Intuitively, the higher an investor's aversion to alpha risk, the lower their active allocation. The importance of alpha risk tolerance for portfolio construction is not new. For instance, Flood and Ramachandran (2000) highlight how the active-passive

Figure 4. Alpha expectation and alpha risk



Notes: For illustrative purposes only. The size of the area representing the probability of the active fund underperforming its passive counterpart (dark blue area) is ultimately also a function of the associated level of alpha risk (i.e., tracking error).
Source: Vanguard.

decision is a risk-budgeting problem, while Waring et al. (2000) and Waring and Siegel (2003) provide a more quantitative framework to find the optimal allocation toward active in a portfolio, assuming that active excess returns and passive returns are independent.

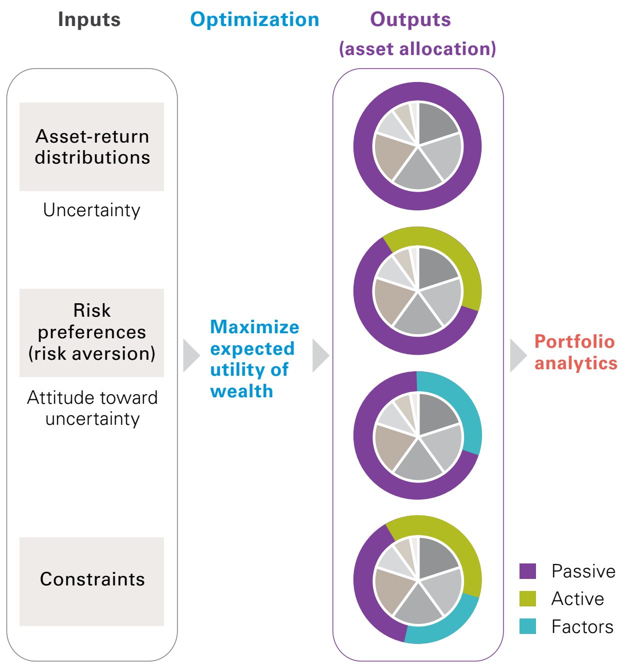
The Vanguard Asset Allocation Model

The VAAM is an expected utility-based model that assesses the risk and return trade-offs of all possible portfolio combinations that meet certain constraints or guardrails.

A utility function helps assess the risk-return trade-off between expected return and uncertainty. In the context of the model, this utility function is a mathematical representation of an investor's attitude toward risk; it translates a stream of expected returns (or, equivalently, an expected level of wealth) into a utility score that is consistent with the investor's attitude toward risk. In effect, the utility function applies a penalty for expected volatility that is dependent on the risk aversion of an investor. For any given set of returns, the more averse to risk the investor is, the higher the applied penalty will be—and the higher the penalty, the more conservative the resulting portfolio will be.

4 Specifically, the distribution of potential risk-adjusted excess returns for that manager.
5 Notable exceptions to this approach are Flood and Ramachandran (2000), Waring et al. (2000), and Waring and Siegel (2003).

Figure 5. An overview of the Vanguard Asset Allocation Model



Note: For illustrative purposes only.
Source: Vanguard.

Figure 5 provides an overview of how the VAAM optimizes a portfolio by identifying the asset allocation that will produce the risk-return trade-off that is most favorable to the investor based on his or her aversion to investment risk, which is mathematically represented by a utility function. The left side of the figure presents the set of inputs used by the model:

- Asset-return distributions (expected returns, volatility, and correlations for all asset classes, active strategies, and long-only factors).
- The investor's attitude toward risk (their systematic risk aversion, alpha risk aversion, and risk aversion to factor premia).
- The investor's portfolio constraints or guardrails.

Generating asset returns distributions for the VAAM

The VCMM is a financial simulation engine that forecasts a distribution of asset returns, volatilities, and correlations for passive assets and factors (see Davis et al., 2014). The VAAM leverages the distributional forecasting framework of the VCMM and benefits from all the features embedded in it, such as sensitivity to initial valuations, non-normal distributions (that is, fatter distribution tails), capturing serial correlation in addition to cross-asset correlation, and accounting for the important linkages between asset returns and the economy (Davis et al., 2018).

While the VCMM generates the return distributions for different asset classes as well as long-only factor benchmarks, the VAAM runs a separate simulation engine for the specific active strategies under consideration for use in the portfolio. This is one of the distinctive features of our model. Relying only on point estimates for the manager's factor-adjusted alpha would ignore the possibility of the fund under-performing its factor-adjusted benchmark. Without risk of underperformance, there would be no risk-return trade-off for the active-passive decision facing the investor.

The active fund simulation engine starts with the factor attribution analysis of the active fund returns. The VAAM then simulates a distribution of expected factor-adjusted alphas using the standard error of the residual—that is, the factor-adjusted tracking error of the manager—in the factor attribution regression. Specifically, the model uses Monte-Carlo methods to simulate a non-normal distribution (*t*-distribution) of factor-adjusted excess returns.

Expected utility-based optimization and investors' attitude toward risk

Typically, risky assets (such as equities) have higher expected returns and higher risk than more conservative assets (such as investment-grade fixed income). An investor's aversion toward risky asset classes is called "systematic risk aversion" in our model.

Investors can exhibit a low level of aversion to systematic risk while simultaneously being risk averse toward alpha. An institutional investor, for example, may have a policy portfolio that targets a large allocation to equity but does not allow for much alpha risk. Such investors are said to have higher alpha risk aversion. As noted earlier, the VAAM incorporates a penalty for alpha risk aversion into the utility scoring, whereby the investor's alpha risk aversion is applied to the factor-adjusted alpha simulation.

Similarly, a penalty for factor risk aversion is applied to the relevant factor premia (defined as excess return over the market) in the portfolio. The VAAM keeps track of all factor exposures in the portfolio—the factor styles of the active managers, as well as the direct factor exposures stemming from allocation to stand-alone long-only factor vehicles such as factor exchange-traded funds (ETFs).

The total expected utility score of a portfolio is the sum of the expected utility scores for systematic risk, alpha risk, and factor risk. The portfolio that results in the highest total expected utility score is considered optimal. More precisely, the VAAM solves for optimal portfolios by maximizing the expected utility of wealth at maturity while penalizing portfolios with higher risk (see **Figure 6**, on page 8). This means that our model computes the average of all utility scores across the total portfolio return distribution (VCMM market and factor simulations plus alpha risk simulations) for each potentially optimal portfolio. In this way, the VAAM is able to trade off between active, passive, and factor allocation simultaneously—while also accounting for alpha risk, investor preferences toward uncertainty (systematic, alpha, and factor risk aversions), uncertainty of returns, and correlations among asset returns.[6]

The VAAM and portfolio constraints

Investors commonly use portfolio constraints to express their portfolio beliefs and reduce the risk of creating portfolios where the optimal weight to one or more of the asset classes is zero (that is, they want to avoid corner solutions). These constraints often take the form of upper and/or lower bounds for exposure (for example, REITs cannot be more than 10% of the total asset allocation, or credit exposure is capped at 50% of the total fixed income allocation) and home bias (for example, U.S. equities must account for at least 60% of the total equity allocation). Our model allows for linear constraints to be taken into account. Also, because the VAAM considers long-only investments, the optimal portfolio weights it generates will always be positive.

Expected utility maximization and calibrating risk aversion

Our model selects the asset allocation strategy that maximizes the expected utility of an investor's wealth at the end of a given investment period (ten years, for example). Expected utility maximization approaches to asset allocation have been used for a long time. Adler and Kritzman (2007) and Sharpe (2007) provide compelling evidence for the benefits of adopting expected utility over mean-variance for portfolio optimization. The VAAM uses a power utility function to model an investor's preference and attitude toward risk.

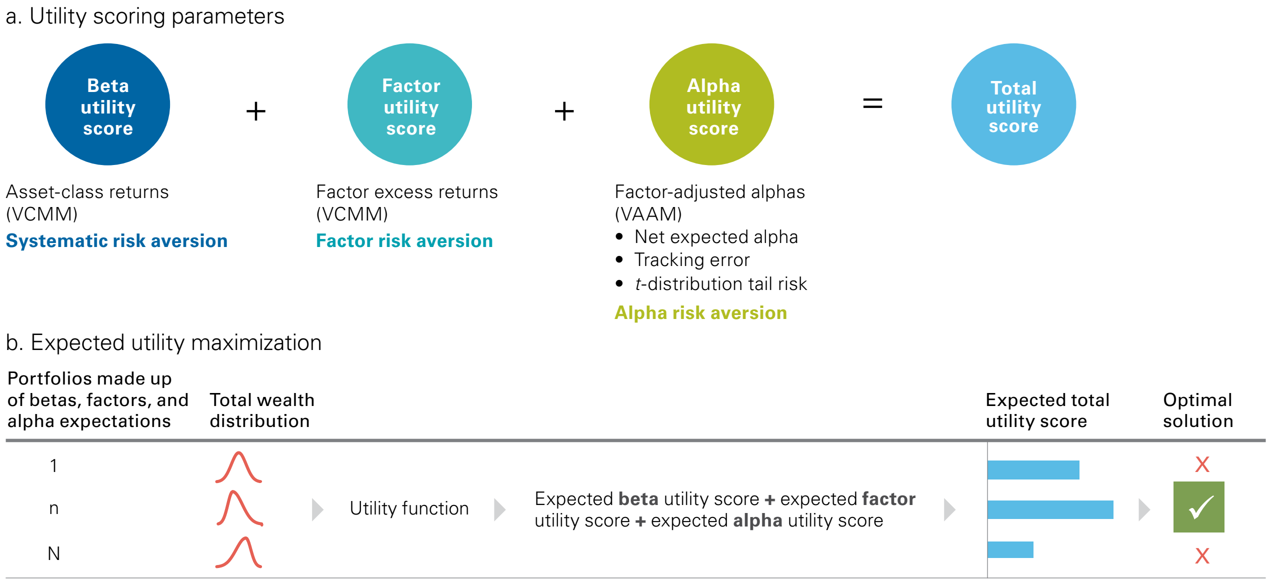
Research focusing on estimating relative risk aversion has been conducted for more than 30 years; more recently, experiments and survey responses have also been used (for example, Metrick, 1995, and Barsky et al., 1997).

In our framework, relative risk aversion coefficients do not have any intuitive economic meaning. Moreover, risk aversion coefficients are ordinal numbers rather than cardinal. This means that an investor who is twice as risk averse as another will show a higher risk aversion coefficient—but not necessarily one that is twice as big.

How shall we approach the challenge of calibrating risk aversion for portfolio optimization studies? To our knowledge, literature on this topic is scarce. To calibrate risk aversion to any type of utility function, non-parametric return distribution, and level of portfolio optimization complexity (for example, multiasset optimization with constraints), Liu and Xu (2010) propose the "efficient frontier" approach. This method creates multiple optimal portfolios using a fairly wide range of initial risk aversion coefficients. Investors would then identify those portfolios that meet their preferences based on a set of risk and performance statistics (expected return, volatility, Sharpe ratio, maximum drawdown, etc.) and keep adjusting the implied risk aversion until the optimal portfolio statistics converge to the desired set of portfolio risk and return metrics.

6 The VAAM incorporates a genetic algorithm to efficiently solve for the optimal portfolio, rather than scoring every possible allocation combination, as that can become cumbersome once the number of assets is large. See Appendix A for further details.

Figure 6. Optimal portfolio selection



Note: For illustrative purposes only.
Source: Vanguard.

The mathematics of the VAAM's expected utility maximization

The VAAM uses a power utility function to model investors' preference and attitude toward risk:

$$U(W) = \begin{cases} \dfrac{W^{1-\gamma}}{1-\gamma}, & \gamma > 1 \\ \ln(W), & \gamma = 1 \end{cases}$$

where γ is the relative risk aversion (RRA) coefficient and W is the level of terminal wealth relative to starting wealth.

Consider an investor facing the portfolio choice problem presented in investing their wealth over an investment horizon. Wealth will compound in each period at the total multiasset portfolio return R_t:

$$R_t = \sum_{i=1}^{N} x_i r_{i,t} = \sum_{i=1}^{N} x_i^p r_{i,t}^p + \sum_{i=1}^{N}\sum_{f=1}^{F} x_i^f r_{i,t}^f + \sum_{i=1}^{N} x_i^a r_{i,t}^a$$

$$\begin{cases} r_{i,t}^p = r_{i,t}^M \\ r_{i,t}^f = r_{i,t}^M + \delta_{i,t}^f \\ r_{i,t}^a = \alpha_i + \beta_i r_{i,t}^M + \sum_{f=1}^{F} L_i^f \delta_{i,t}^f + \varepsilon_{i,t} \qquad \varepsilon_{i,t} \sim t(v)\sqrt{\sigma_{\alpha_i}^2} \end{cases}$$

where x_i and r_i are the portfolio weights and relative total returns for each asset class i and superscripts p, f, and a refer to passive, factors, and active, respectively.

The market benchmark return is represented by $r_{i,t}^M$; $\delta_{i,t}^f$ is the excess (to the market benchmark) factor return for factor f; and β_i and L_i correspond to, respectively, the market beta and factor loading. α_i is the pure (that is, factor-adjusted) excess active return.

The portfolio choice problem consists of finding optimal weights for each passive asset class, factor, or active manager or strategy in the portfolio. We then express the expected utility-based optimization problem that we want to solve for as:

$$\max_{x} \mathbb{E}\left[U\left(\frac{W_T}{W_0}\right)\right] \rightarrow \max_{x} \left\{ \mathbb{E}\left[\frac{W_p^{1-\gamma_p}}{1-\gamma_p}\right] + \mathbb{E}\left[\frac{W_f^{1-\gamma_f}}{1-\gamma_f}\right] + \mathbb{E}\left[\frac{W_a^{1-\gamma_a}}{1-\gamma_a}\right] \right\}$$

$$s.t.\ \{x_i \in \mathbb{R} \mid 0 \le x_i \le 1\} \wedge \sum_i x_i = 1$$

$$\sum_i C \cdot x_i \le b$$

where W_p, W_f, and W_a are the wealth at maturity coming from systematic, factor, and factor-adjusted alpha exposures, respectively, γ_p, γ_f, and γ_a are the systematic, factor, and alpha risk aversions, respectively, and C and b refer to the set of linear inequality constraints.

How does the VAAM respond to custom investor inputs?

The driving force behind the creation of the VAAM has been the desire to deliver customized portfolios to investors based on their risk preferences. As we explained earlier, this requires simultaneously managing multiple trade-offs investors face when building their portfolio.

Before diving into the other specific applications of the VAAM, it's helpful to consider how the model's explicit inputs affect the portfolio asset allocation. For instance, all else being equal, one would expect that lowering systematic risk aversion—and thus reducing the penalty toward dispersion of asset-class returns—would increase equity allocation in a multiasset portfolio. To demonstrate this, we provided the VAAM with several systematic risk aversion coefficients as input, solved for optimal portfolios (keeping alpha and factor risk aversion constant), and observed the total equity allocation as the risk aversion was decreased (**Figure 7a**).

By design, lowering alpha risk aversion (decreasing the penalty toward dispersion of factor-adjusted alpha distribution) should increase active allocation. **Figure 7b** indeed confirms the intuition that allocation to a U.S. equity active fund as a percentage of total U.S. equity allocation (active and passive) increases as the alpha risk aversion is reduced.[7] Similarly, **Figure 7c** demonstrates that reducing factor risk aversion increases a portfolio's allocation to factors. In this example, we assess the allocation of U.S. equity passive factor vehicles relative to total U.S. equity allocation (active, passive, and factor) by keeping the systematic and alpha risk aversion constant.

What is the impact of lowering factor-adjusted alpha expectations? **Figure 8**, on page 10, presents four portfolios. Two of them—Portfolios A and B—have identical inputs aside from their gross factor-adjusted alpha, which is 81 bps for Portfolio A and 11 bps for Portfolio B. Lowering factor-adjusted alpha reduces the allocation to the U.S. equities active fund from 30% to 16%.

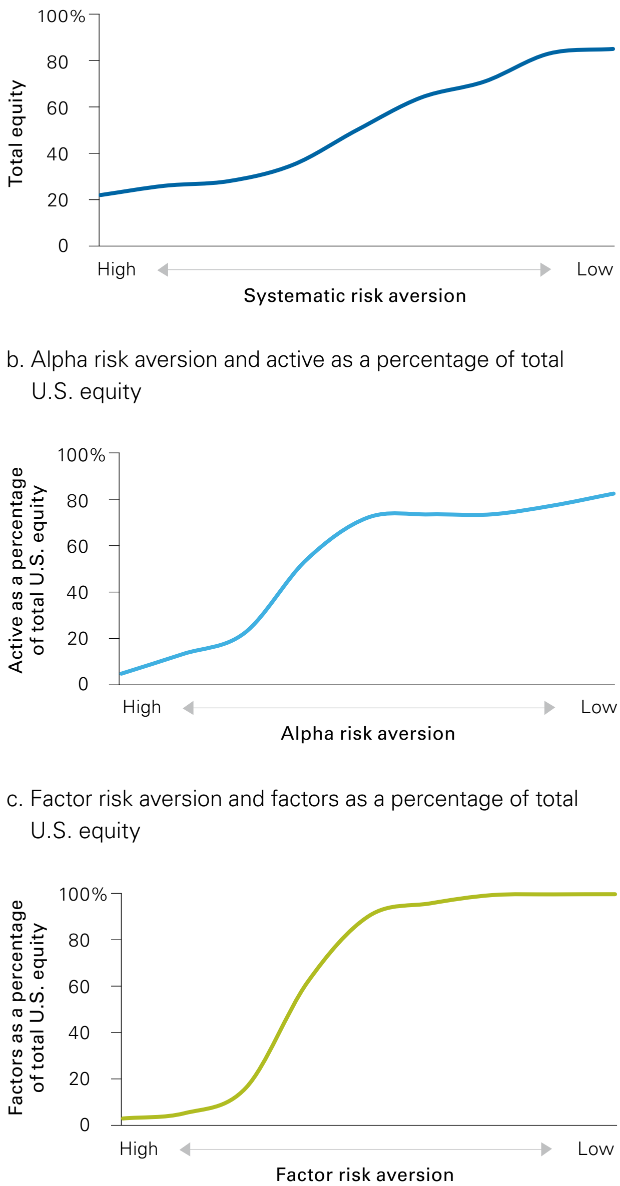
We also checked the impact of lowering dispersion around the factor-adjusted alpha. Portfolios C and D in Figure 8 show how in this case the allocation toward active increases. Here, we lowered the dispersion parameter (i.e., tracking error) of the active fund reported in Figure 3 from 4.03% to 2.00% and increased the level of alpha risk aversion, so we could compare two portfolios

Figure 7. Asset allocation and risk aversion levels

a. Systematic risk aversion and total equity allocation

b. Alpha risk aversion and active as a percentage of total U.S. equity

c. Factor risk aversion and factors as a percentage of total U.S. equity



Notes: Portfolios have been optimized over a ten-year investment horizon using U.S. equities, non-U.S. equities, U.S. bonds, non-U.S. bonds, intermediate-term U.S. credit bonds, and short-term U.S. credit bonds. Non-U.S. bonds are hedged to USD. The following constraints apply: non-U.S. equities, up to 40% of the total equity allocation; non-U.S. bonds, up to 50% of total (non-credit) bonds; total credit bonds, up to 50% of total fixed income (bonds and credit bonds); intermediate-term U.S. credit bonds, up to 60% of total credit bonds; short-term U.S. credit bonds, up to 60% of total credit bonds. Active and factor allocation options are considered for U.S. equities only. Market beta, factor loadings, expected alpha, and tracking error estimates for the U.S. equities active fund are shown in Figure 3.

Sources: Vanguard calculations, using asset-return projections from the VCMM as of June 30, 2018.

7 The active fund used is the same as the one reported in Figure 3. In all the examples reported in this paper, we consider the option to include exposure toward alpha risk and factor risk in the portfolio for U.S. equities only.

Figure 8. Optimized asset allocations

Risk aversion	Portfolio A	Portfolio B	Portfolio C	Portfolio D
Systematic	Medium	Medium	Medium	Medium
Alpha	Low	Low	Medium	Medium
Factor	Medium	Medium	Medium	Medium
Active risk characteristics				
Factor-adjusted expected alpha	81 bps	11 bps	81 bps	81 bps
Tracking error	4.03%	4.03%	4.03%	2.00%
Asset-class weights				
U.S. equities	40%	39%	43%	39%
Passive	*6%*	*15%*	*22%*	*9%*
Active	*30%*	*16%*	*13%*	*26%*
Value factor	*3%*	*5%*	*5%*	*4%*
Mid-cap factor	*1%*	*3%*	*3%*	*0%*
Non-U.S. equities	25%	25%	26%	26%
U.S. bonds	19%	21%	17%	24%
Non-U.S. bonds	1%	1%	3%	1%
Intermediate-term U.S. credit bonds	8%	8%	5%	5%
Short-term U.S. credit bonds	7%	6%	6%	5%
	100%	100%	100%	100%
Summary statistics				
Total equity allocation	65%	64%	69%	65%
Expected return	5.6%	5.3%	5.3%	5.5%
Expected volatility	10.5%	10.3%	10.9%	10.4%

Notes: Portfolios have been optimized over a ten-year investment horizon using U.S. equities, non-U.S. equities, U.S. bonds, non-U.S. bonds, intermediate-term U.S. credit bonds, and short-term U.S. credit bonds. Non-U.S. bonds are hedged to USD. The following constraints apply: non-U.S. equities, up to 40% of the total equity allocation; non-U.S. bonds, up to 50% of total (non-credit) bonds; total credit bonds, up to 50% of total fixed income (bonds and credit bonds); intermediate-term U.S. credit bonds, up to 60% of total credit bonds; short-term U.S. credit bonds, up to 60% of total credit bonds. Market beta, factor loadings, and tracking error estimates for the U.S. equities active fund are as reported in Figure 3. Factor-adjusted expected alphas are assumed to be before fees.

Sources: Vanguard calculations, using asset-return projections from the VCMM as of June 30, 2018.

with significant difference in the active allocation. When evaluating these results, it is important to recognize the significant increase in the active manager's skill, as measured by the information ratio. In this example, by virtue of the tracking error being reduced, the manager's excess return per unit of risk (i.e., the information ratio) has increased from 0.20 (81/403 bps) to 0.40 (81/200 bps).

Model applications

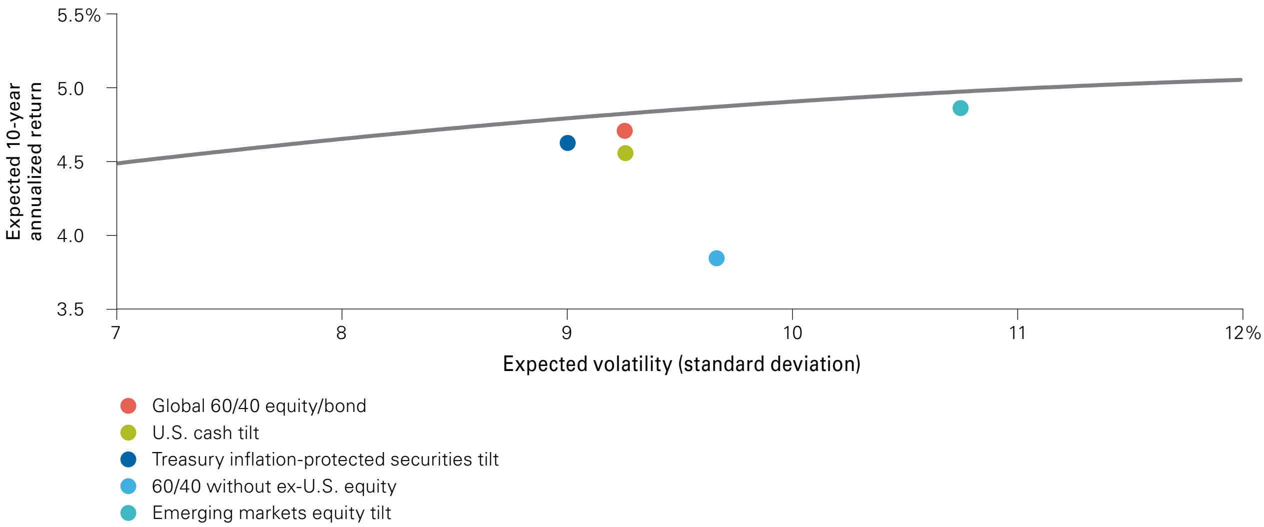
The VAAM can have multiple research and business applications. It can be used to deliver solutions for investors globally, such as for strategic factor model portfolios, in the active-passive decisions and bespoke asset allocation. It can also be used by investors to replace high-cost active with lower-cost factors in their portfolio.

Passive fund/ETF model portfolios

For an investor looking to get exposure to the markets through passive funds or ETFs, the VAAM can be used to deliver a traditional passive-only asset allocation. This case is simply a subset of the cases the model can handle. Investors have two levers: They can specify their opportunity set—the basket of the different passive investments and/or asset classes to be considered—and they can specify their risk preferences by choosing their level of systematic risk aversion and muting their options in loading active and factor investments.

The efficient frontier, shown as a gray line in **Figure 9**, is constructed following this approach. The dots in the figure represent portfolios with popular ad hoc tilts. In attempting to increase portfolio returns, investors might decide to overweight high-yield assets such as emerging markets equities. Alternatively, investors might introduce cash tilts in their portfolios to mitigate risk. The problem with such ad hoc tilts is that they ignore correlations among assets and can lead to inefficient portfolios. Portfolio tilts should be assessed within an optimization framework. The efficient frontier in Figure 9, which is constructed using traditional asset classes only, still lies above all other ad hoc portfolios, and thus illustrates the value added through an optimization approach.

Figure 9. Portfolio tilts should be assessed within an optimization framework



Notes: For the efficient frontier, shown as a gray line in the figure, portfolios have been optimized over a ten-year investment horizon using U.S. equities, non-U.S. equities, U.S. bonds, non-U.S. bonds, intermediate-term U.S. credit bonds, and short-term U.S. credit bonds. Non-U.S. bonds are hedged to USD. The following constraints apply: non-U.S. equities, up to 40% of the total equity allocation; non-U.S. bonds, up to 50% of total (non-credit) bonds; total credit bonds, up to 50% of total fixed income (bonds and credit bonds); intermediate-term U.S. credit bonds, up to 60% of total credit bonds; short-term U.S. credit bonds, up to 60% of total credit bonds. The equity portion of the global 60/40 equity/bond portfolio is 60% U.S. equity and 40% global ex-U.S. equity; the bond portion of the portfolio is 70% U.S. bonds and 30% global ex-U.S. bonds. Portfolios with tilts include a 20% tilt from the global 60/40 equity/bond portfolio to the asset specified, with the fixed income tilts funded from the fixed income allocation and the equity tilts funded from the equity allocation.

Sources: Vanguard calculations, using asset-return projections from the VCMM as of June 30, 2018.

Factor investing in the context of a multiasset portfolio

A significant portion of the research done on factor investing focuses on stand-alone strategies without considering how factors play a role in a multiasset portfolio. As highlighted by Fisher and McDonald (2018), little research has been done on how to include factors as part of an optimal portfolio based on risk preferences. Bergeron, Kritzman, and Sivitsky (2018) propose an approach for asset allocation and factor investing where these can be combined to achieve a portfolio sensitive to the desired factor profile. The research conducted by Rao, Subramanian, and Melas (2018) is probably the best example to date of combining active, factors, and passive in a single solution, albeit in the equity space only. The VAAM expands on current research done on this topic and provides an approach that allows us to simultaneously solve for passive, active, and factor allocation in the multiasset space, given a set of investor risk aversions toward systematic, alpha, and factor risks.[8]

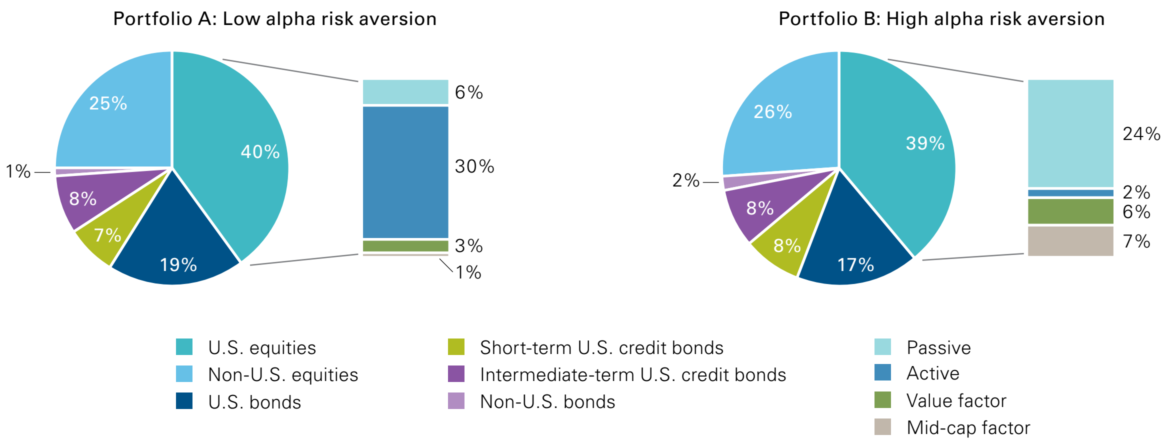
Factor model portfolios

For an investor who doesn't want any active in their portfolio but would consider factor-tilted portfolios, the model can provide an exposure to available asset classes and factors. The factor exposures can be obtained by investing in indexed factor vehicles such as factor mutual funds or ETFs. By specifying different levels of factor risk aversion, the model can construct optimal portfolios with factor tilts that take into account the forecasted risk-return characteristics of each factor along with their cross-correlations.

Manager substitution analysis

How can we account for the case where an investor wants to get exposure toward the implicit factors that would come with an active strategy but does not want to bear the additional pure (factor-adjusted) alpha risk? This scenario can be modeled by keeping the factor risk aversion reasonably low and increasing the alpha risk aversion. In this way the model should tell us how much of the implicit factor exposure that comes with the active strategy can be replicated passively through passive factor investment vehicles such as factor ETFs. **Figure 10** shows an example.

Figure 10. Substitution effect



	Risk aversion			Summary statistics	
Case Study	Systematic	Alpha	Factor	Expected return	Expected volatility
Portfolio A	Medium	Low	Medium	5.6%	10.5%
Portfolio B	Medium	High	Medium	5.3%	10.4%

Notes: Portfolios have been optimized over a ten-year investment horizon using U.S. equities, non-U.S. equities, U.S. bonds, non-U.S. bonds, intermediate-term U.S. credit bonds, and short-term U.S. credit bonds. Non-U.S. bonds are hedged to USD. The following constraints apply: non-U.S. equities, up to 40% of the total equity allocation; non-U.S. bonds, up to 50% of total (non-credit) bonds; total credit bonds, up to 50% of total fixed income (bonds and credit bonds); intermediate-term U.S. credit bonds, up to 60% of total credit bonds; short-term U.S. credit bonds, up to 60% of total credit bonds. Market beta, factor loadings, and tracking error estimates for the U.S. equities active fund are reported in Figure 3.

Sources: Vanguard calculations, using asset-return projections from the VCMM as of June 30, 2018.

8 Waring et al. (2000) introduced, in an expected utility maximization setting, the importance of finding an optimal asset allocation and active exposure all at once, as the study recognized that most of the time investors accomplish the task in steps rather than simultaneously, which is suboptimal.

Portfolio A in this figure is identical to Portfolio A in Figure 8 and is presented for comparison purposes. Portfolio B has the same factor risk aversion as Portfolio A—but it also incorporates a high level of alpha risk aversion. Increasing the alpha risk aversion leads to a decrease in the active allocation and a simultaneous increase in the indexed factor allocation. This is precisely the substitution effect, from implicit factor exposure via active allocation to explicit indexed factor allocation, that we were looking for. Put another way: The potentially high-cost active fund is replaced with lower-cost passive factors.

VAAM portfolios under different initial conditions

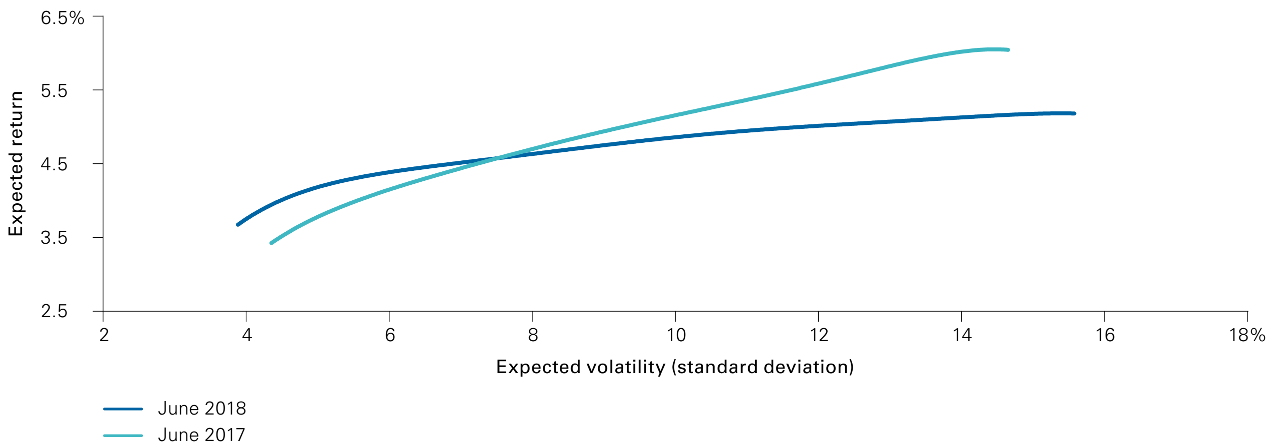
The VAAM uses asset-return projections from the VCMM as an input in order to create optimal portfolios. As mentioned above, VCMM forecasts take into account initial conditions such as current equity valuations and short-term interest rates. Even if long-term (i.e., steady state) assumptions and forecasts do not change, any optimal portfolio constructed over an investment horizon that includes short- and medium-term projections will be affected by current market conditions and changes. **Figure 11** shows this feature and how the VAAM efficient frontier changed over the course of one year.

VAAM portfolios under different economic scenarios

Similarly, the VAAM can be used to construct portfolios that are optimized for some specific economic scenarios and environments. **Figure 12**, on page 14, presents three optimal portfolios based on three different economic scenarios: central case, recession, and high growth.

The high-growth scenario illustrates an upside risk scenario of sustained economic growth. The two others are our central-case scenario driven by moderate volatility with positive financial conditions—this corresponds to the VCMM central forecast—and a recessionary scenario caused by a turn in the business cycle and a correction in the equity markets. In a high-growth scenario, expected global equity returns would be high. Long and short rates would also rise faster than expected, resulting in an optimal portfolio loading on equity. A recessionary-scenario portfolio would underweight equity and overweight bonds with longer durations. The portfolio strategy in a central-case scenario is well-diversified.

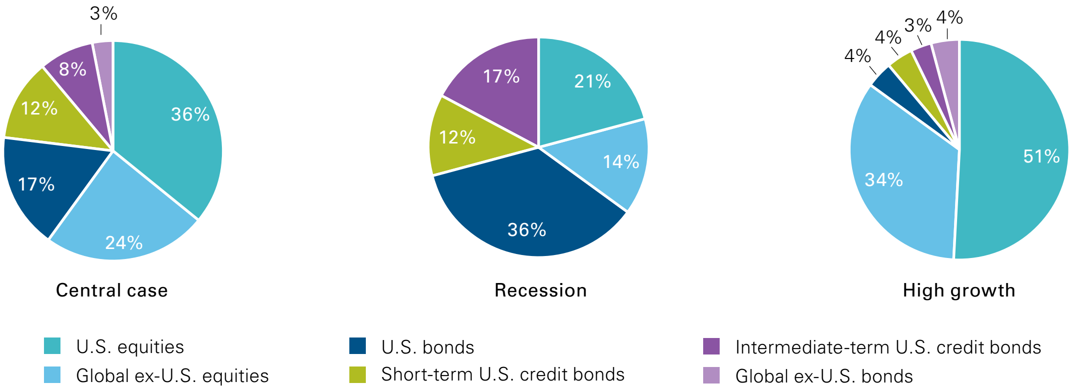
Figure 11. VAAM efficient frontiers at different points in time



Notes: Portfolios have been optimized over a ten-year investment horizon using U.S. equities, non-U.S. equities, U.S. bonds, non-U.S. bonds, intermediate-term U.S. credit bonds and short-term U.S. credit bonds. Non-U.S. bonds are hedged to USD. The following constraints apply: non-U.S. equities, up to 40% of the total equity allocation; non-U.S. bonds, up to 50% of total (non-credit) bonds; total credit bonds, up to 50% of total fixed income (bonds and credit bonds); intermediate-term U.S. credit bonds, up to 60% of total credit bonds; short-term U.S. credit bonds, up to 60% of total credit bonds.

Sources: Vanguard calculations, using asset-return projections from the VCMM as of June 30, 2017, and June 30, 2018.

Figure 12. VAAM optimal portfolios for different economic environments



Notes: The forecast displays a simulation of three-year annualized returns of asset classes shown as of June 30, 2018. Scenarios are derived from sorting the VCMM simulations based on rates, growth, volatility, and equity returns. The three scenarios are a subset of the 10,000 VCMM simulations. The central-case portfolio is constructed to target a 60/40 equity/bond split. The recession and high-growth portfolios assume the same risk aversion as the central case and allow the total equity allocation to fluctuate between 35% and 85%. The following constraints apply: non-U.S. equities, up to 40% of the total equity allocation; non-U.S. bonds, up to 50% of total (non-credit) bonds; total credit bonds, up to 50% of total fixed income (bonds and credit bonds); intermediate-term U.S. credit bonds, up to 60% of total credit bonds; short-term U.S. credit bonds, up to 60% of total credit bonds.

Sources: Vanguard calculations, using asset-return projections from the VCMM as of June 30, 2018.

The model's limitations, and some caveats

Like any asset allocation model, the VAAM possesses a set of specific assumptions, limitations, and measurement imprecisions.

Again, we note that the VAAM's output is based on VCMM forecasts that it uses as inputs. VCMM projections are based both on estimated historical relationships and on assumptions about the risk characteristics of the different asset classes. The accuracy of the model's forecasts depends, then, on the relevance of the historical sample used to forecast future events (see Davis et al., 2014). Therefore, if our asset-class return, volatility, and correlation projections are inaccurate, the optimal weights estimated by the VAAM could be biased. Similarly, the factor decomposition (Figure 2) that is performed to estimate the factor-adjusted alpha and tracking error is subject to estimation error, leading to parameter uncertainty.

The VAAM, like any model of its type, provides one simplified representation of reality and the set of complex decisions that investors have to make to find their asset allocation. Therefore, it represents one possible model specification. Different specifications, of the same or similar approach, could be identified as being more suitable for some specific investors (i.e., model uncertainty). For instance, one of the main assumptions in the VAAM is that while they might have very different levels of risk aversion toward systematic, alpha, and factor risk, all investors share the same utility function: a power utility function. Although the power utility function is widely and well accepted in utility maximization problems, it might not represent the actual attitude toward expected wealth for all investors.

Finally, the VAAM uses a genetic algorithm to find the optimal weights (see Appendix A). Genetic algorithms are stochastic methods and therefore the solution they converge to is subject to random sampling noise.

Although our model presents a set of reasonably strong assumptions and limitations, it nevertheless helps in providing a methodology and framework to find optimal asset allocation solutions to multidimensional problems with systematic, alpha, and factor risks.

Conclusion

The VAAM is a proprietary model for allocating assets among active, passive, and factors simultaneously that is driven by uncertainty in active returns and an investor's risk preferences towards that uncertainty. The model leverages the distributional forecasting framework of the VCMM and benefits from the features embedded in it, such as sensitivity to initial valuations, forward-looking capital market equilibrium assumptions, non-normal distributions, the capturing of autocorrelation and cross-asset correlation, and important linkages between asset returns and macroeconomic factors.

The VAAM has multiple research and business applications. From an advice business perspective, one of the main benefits of its quantitative framework is that it can be leveraged across various advice and digital technology platforms. The model allows for full customization of portfolios, while at the same time preserving scalability in mass service offerings through technology implementations and ensuring consistency of the underlying investment methodology across the different portfolios.

From a due-diligence and regulatory perspective, its quantitative approach adds more transparency to the asset allocation process. Whether the model is used on an advice platform or within investment committees, this added transparency leads to more straightforward oversight and review processes for portfolio recommendations. After all, the model's methodological underpinnings are based on well-established theories in the academic literature on portfolio choice and household finance.

From a behavioral investment perspective, there are a few advantages from using the model to solve asset allocation problems. Its quantitative nature brings to light many decisions that investors would otherwise make in a subconscious or implicit way when choosing ad hoc portfolio allocations. The input requirements enable a conversation with investors about the conscious and explicit choices that must be made and are critical to the portfolio, such as setting realistic alpha expectations for the active strategies under consideration, selecting the best estimates for the associated alpha risk, and even reflecting on their own aversion to alpha risk.

A common misconception among practitioners is to think that the challenges involved in providing estimates for alpha expectation (or alpha targets) or investor risk tolerance levels are unique to quantitative asset allocation models such as the VAAM. In reality, any asset allocation decision by an investor, with or without a model, entails making some sort of assumption about alpha and risk tolerance (as well as about the equity risk premium, bond risk premium, risk-free rates, etc.). The only difference between "model-free" portfolios and asset allocation models is that with the latter, such assumptions are explicit, which makes them transparent and thus more able to be scrutinized.

References

Adler, Tim, and Mark Kritzman, 2007. Mean-Variance Versus Full-Scale Optimization: In and Out of Sample. *Journal of Asset Management* 7(5): 302–311.

Barsky, Robert B., Thomas Juster, Miles S. Kimball, and Matthew D. Shapiro, 1997. Preference Parameters and Behavioral Heterogeneity: An Experimental Approach in the Health and Retirement Study. *The Quarterly Journal of Economics* 112(2): 537–579.

Bender, Jennifer, P. Brett Hammond, and William Mok, 2014. Can Alpha Be Captured by Risk Premia? *The Journal of Portfolio Management* 40(2): 18–29.

Bergeron, Alain, Mark Kritzman, and Gleb Sivitsky, 2018. Asset Allocation and Factor Investing: An Integrated Approach. *The Journal of Portfolio Management* 44(4): 32–38.

Chin, Andrew, and Piyush Gupta, 2017. Using Prime Alpha to Separate Skill From Luck in Fixed-Income Strategies. *The Journal of Investing* 26(2): 102–109.

Davis, Joseph, Roger Aliaga-Díaz, Harshdeep Ahluwalia, Frank Polanco, and Christos Tasopoulos, 2014. *Vanguard Global Capital Markets Model*. Valley Forge, Pa.: The Vanguard Group.

Davis, Joseph, Roger Aliaga-Díaz, Harshdeep Ahluwalia, and Ravi Tolani, 2018. Improving U.S. Stock Return Forecasts: A "Fair-Value" CAPE Approach. *The Journal of Portfolio Management* 44(3): 43–55.

Fama, Eugene F., and Kenneth R. French, 1993. Common Risk Factors in the Returns on Stocks and Bonds. *Journal of Financial Economics* 33: 3–56.

Fisher, Gregg S., and Michael B. McDonald, 2018. *Factor Allocation and Asset Allocation. The Journal of Wealth Management* 21(2): 10–20.

Flood, Eugene, and Narayan Ramachandran, 2000. Integrating Active and Passive Management. *The Journal of Portfolio Management* 27(1): 10–19.

Holland, John, 1975. *Adaptation in Natural and Artificial Systems*. Ann Arbor, Mich.: University of Michigan Press.

Kshatriya, Saranya, and Krishna Prasanna, 2017. Genetic Algorithm-Based Portfolio Optimization With Higher Moments in Global Stock Markets. *The Journal of Risk* 20(4): 1–26.

Lin, Chi-Ming, and Mitsuo Gen, 2007. An Effective Decision-Based Genetic Algorithm Approach to Multiobjective Portfolio Optimization Problem. *Applied Mathematical Sciences* 1(5): 201–210.

Lin, Dan, Xiaoming Li, and Minquing Li, 2005. A Genetic Algorithm for Solving Portfolio Optimization Problems With Transaction Costs and Minimum Transaction Lots. In *Advances in Natural Paper Computation,* edited by Lipo Wang, Ke Chen, and Yew Soon Ong. Lecture Notes in Computer Science, vol. 31. Berlin: Springer.

Liu, Scott, and Rong Xu, 2010. *The Effects of Risk Aversion on Optimization.* MSCI Barra Research Paper No. 2010-06.

Markowitz, Harry, 1952. Portfolio Selection. *The Journal of Finance* 7(1): 77–91.

Metrick, Andrew, 1995. A Natural Experiment in "Jeopardy!". *American Economic Review* 85(1): 240–253.

Rao, Anil, Raman Aylur Subramanian, and Dimitris Melas, 2018. Bridging the Gap: Adding Factors to Passive and Active Allocations. *The Journal of Portfolio Management* 44(4): 17–31.

Rifki, Omar, and Hirotaka Ono, 2012. A Survey of Computational Approaches to Portfolio Optimization by Genetic Algorithms. Working Paper, Kyushu University.

Roberts, Daren, Thomas Paradise, and Chris Tidmore, 2018. *Global Active Bond Fund Returns: A Factor Decomposition.* Valley Forge, Pa.: The Vanguard Group.

Rowley, James J., Jr., Garrett Harbron, and Matthew C. Tufano, 2017. *In Pursuit of Alpha: Evaluating Active and Passive Strategies.* Valley Forge, Pa.: The Vanguard Group.

Sharpe, William F., 1991. The Arithmetic of Active Management. *Financial Analysts Journal* 47(1): 7–9.

Sharpe, William F., 1992. Asset Allocation: Management Style and Performance Measurement. *The Journal of Portfolio Management* 18(2): 7–19.

Sharpe, William F., 2007. Expected Utility Asset Allocation. *Financial Analysts Journal* 63(5): 18–30.

Wallick, Daniel W., Brian R. Wimmer, Christos Tasopoulos, James Balsamo, and Joshua M. Hirt, 2017. *Making the Implicit Explicit: A Framework for the Active-Passive Decision.* Valley Forge, Pa.: The Vanguard Group.

Waring, Barton M., Duane Whitney, John Pirone, and Charles Castille, 2000. Optimizing Manager Structure and Budgeting Manager Risk. *The Journal of Portfolio Management* 26(3): 90–104.

Waring, Barton M., and Laurence B. Siegel, 2003. The Dimensions of Active Management. *The Journal of Portfolio Management* 29(3): 35–51.

About the Vanguard Capital Markets Model

IMPORTANT: The projections and other information generated by the Vanguard Capital Markets Model (VCMM) regarding the likelihood of various investment outcomes are hypothetical in nature, do not reflect actual investment results, and are not guarantees of future results. VCMM results will vary with each use and over time.

The VCMM projections are based on a statistical analysis of historical data. Future returns may behave differently from the historical patterns captured in the VCMM. More important, the VCMM may be underestimating extreme negative scenarios unobserved in the historical period on which the model estimation is based.

The VCMM is a proprietary financial simulation tool developed and maintained by Vanguard's primary investment research and advice teams. The model forecasts distributions of future returns for a wide array of broad asset classes. Those asset classes include U.S. and international equity markets, several maturities of the U.S. Treasury and corporate fixed income markets, international fixed income markets, U.S. money markets, commodities, and certain alternative investment strategies. The theoretical and empirical foundation for the VCMM is that the returns of various asset classes reflect the compensation investors require for bearing different types of systematic risk (beta). At the core of the model are estimates of the dynamic statistical relationship between risk factors and asset returns, obtained from statistical analysis based on available monthly financial and economic data from as early as 1960. Using a system of estimated equations, the model then applies a Monte Carlo simulation method to project the estimated interrelationships among risk factors and asset classes as well as uncertainty and randomness over time. The model generates a large set of simulated outcomes for each asset class over several time horizons. Forecasts are obtained by computing measures of central tendency in these simulations. Results produced by the tool will vary with each use and over time.

Appendix A. Genetic algorithms in portfolio optimization

Standard mean-variance portfolio optimization problems with linear constraints can be solved using quadratic programming. However, once nonlinear constraints, such as transaction costs or minimum lots in the portfolio (Lin, Li, and Li, 2005), or higher moments (i.e., skewness and kurtosis) are captured by the distribution of the forecasts (Kshatriya and Prasanna, 2017), the optimization problem becomes non-convex and computationally unfeasible. In those instances, a "derivative-free" approach such as a genetic algorithm can become very valuable.

Originally developed by Holland (1975), genetic algorithms are stochastic methods inspired by natural selection processes and are now widely used for constrained and unconstrained portfolio optimization problems. For example, Rifki and Ono (2012) provide a comprehensive literature review on computational approaches to portfolio optimization using genetic algorithms. Compared with more traditional optimization methods, genetic algorithms present the advantage of requiring little or no knowledge of the problem (i.e., search space) at hand. Also, they can perform well in large, complex, and multi-objective problems (Lin and Gen, 2007) and are more likely to converge to the global (as opposed to local) optima of the problem. For these reasons, the optimized portfolio weights presented in our analysis are computed using a genetic algorithm.

Appendix B. U.S. equity style factor definitions

The table below shows the criteria that were used to define and construct U.S. equity style factors for the analysis reported in this paper. VAAM methodology is not dependent on the definition of factors shown below, and the model can accommodate any other factor definition or benchmark.

Factor	Data start point	Succinct definition	Selection universe	Weighting scheme
Value	January 1980	1/3 of stocks with the lowest price-to-book ratio	Russell 1000 Index	Market-capitalization-weighted
Growth	January 1980	1/3 of stocks with the highest price-to-book ratio	Russell 1000 Index	Market-capitalization-weighted
Large-cap	January 1980	2/3 of stocks with the highest market capitalization	Russell 1000 Index	Market-capitalization-weighted
Mid-cap	January 1980	1/3 of stocks with the lowest market capitalization	Russell 1000 Index	Market-capitalization-weighted
Small-cap	January 1980	2/3 of stocks with the lowest market capitalization	Russell 3000 Index	Market-capitalization-weighted
Momentum	January 1980	1/3 of stocks with the highest 12-month trailing returns	Russell 1000 Index	Market-capitalization-weighted
Low volatility	January 1980	2/10 of stocks with the lowest annualized return volatility	Russell 1000 Index	Inverse of volatility
Quality	January 1987	1/3 of stocks with the highest quality score	Russell 1000 Index	*Market capitalization × quality score* *Quality score = (profitability score + investment score)* / 2
Liquidity	January 1992	1/3 of stocks with the highest illiquidity score	Russell 3000 Index	*Market capitalization × illiquidity score* *Illiquidity score = (share turnover + dollar turnover + Amihud illiquidity)* / 3

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