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Focus on the main thing: Maximizing capital efficiency in liability-hedging fixed income

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- When designing a liability-driven investing (LDI) strategy, understanding and prioritizing the main drivers of pension liability risk could lead to more efficient surplus risk mitigation, particularly if liability-hedging capital is limited.

- The key drivers of liability risk can be measured along two dimensions: projected benefit payment date (near-term versus long-dated) and drivers of discount rate changes (interest rate, credit spreads, and yield curve shape changes).

- We find that most risk is associated with the longest-dated benefit payments and with discount rate changes attributable to parallel Treasury yield curve changes.

- These findings indicate that a plan sponsor's liability-hedging strategy should focus primarily on the longest-dated benefit payments, parallel shifts in the Treasury yield curve, and movements in credit spreads.

Introduction

Liability-driven investment (LDI) strategies have become increasingly common among corporate pension plan sponsors. A full 75% of respondents to the 2018 Vanguard pension sponsor survey have implemented some sort of LDI strategy to mitigate surplus risk (the risk of unexpected changes in the plan's surplus or deficit relative to its liability).

An essential component of every LDI strategy is the *liability-hedging* portfolio. This is the portion of plan assets designed specifically to mirror the liability's risk profile in order to generate returns highly correlated with liability movements.[1] While designed to mitigate *surplus risk*, the liability-hedging portfolio can never eliminate all of that risk. A number of non-hedgeable risks are embedded in the measurement of pension liabilities (this is discussed in more detail in Bosse, 2018).[2] Furthermore, the value of assets available for liability hedging is often less than that of the plan's liability.[3] Many pension sponsors find themselves in this situation because their plan is underfunded, part of its investment portfolio is dedicated to *return-seeking* assets (assets such as equities, with the potential to grow faster than a pension liability over the long term), or often for both reasons.

When the amount of liability-hedging assets is less than the amount of pension liability, a plan sponsor must make an intentional decision about how to deploy these assets.

1. **Keep duration shorter than that of the liability.** This strategy has typically been adopted by plan sponsors who espouse a total-return approach (non-asset-liability view) to pension investing, have yet to be introduced to the benefits of an LDI strategy and are still invested in an aggregate fixed income benchmark, or have a market-timing view that interest rates will rise and therefore the portfolio will benefit from being under-hedged.
2. **Target the liability's duration and yield curve profile.** This strategy distributes the portfolio proportionately relative to the plan's benefit payment pattern or the liability's key rate duration profile, providing a consistent hedge ratio across the entire maturity range. This allows easy measurement of how effectively an LDI program tracks the liability by comparing the returns of the liability-hedging portfolio with the return of the liability itself. The focus of the strategy is to lower liability tracking error (the difference between the two returns). However, this strategy might still leave a pension plan significantly under-hedged with respect to interest rates and therefore may not be very effective at minimizing total

Notes on risk

All investing is subject to risk, including possible loss of principal.

Past performance does not guarantee future results.

Bond funds are subject to interest rate risk, which is the chance bond prices overall will decline because of rising interest rates, and credit risk, which is the chance a bond issuer will fail to pay interest and principal in a timely manner or that negative perceptions of the issuer's ability to make such payments will cause the price of that bond to decline.

U.S. government backing of Treasury or agency securities applies only to the underlying securities and does not prevent share-price fluctuations. Unlike stocks and bonds, U.S. Treasury bills are guaranteed as to the timely payment of principal and interest.

There is no guarantee that any particular asset allocation or mix of funds will meet your investment objectives or provide you with a given level of income.

Diversification does not ensure a profit or protect against a loss.

1 Best practices for constructing liability-hedging portfolios are discussed in several recent Vanguard papers, including Bosse (2018) and Dutton and Plink (2018).

2 One might think there are investments that can hedge 100% of the plan liability whether from the point of view of payment timing or the source of discount rate movements. We do not make this claim. Asset liability risk will remain for a variety of reasons, including actuarial assumption risk related to demographics, economics, and basis risk created by the methodology used to construct the liability yield curve. Yield curves cannot be bought or managed and are constructed to ignore period-to-period downgrades and defaults. The idea of hedging 100% of a plan liability with investments is purely theoretical.

3 In this circumstance, derivative overlays can help mitigate surplus risk, but they are less effective than additional physical liability-hedging assets. Because of the cost and complexity of implementing a derivative overlay strategy, its usage is not common today. The 2018 Vanguard pension sponsor survey reports that 25% of respondents use derivatives for liability-hedging purposes.

surplus risk. This is especially true when there is a low allocation to the liability-hedging portfolio, a low funded status, or (typically) both.

3. **Keep duration longer than that of the liability.** Finally, for a more effective interest rate hedge and a more efficient use of limited assets, the liability-hedging portfolio could be constructed to offset the duration risk in the longest-dated benefit payments. This typically results in a fixed income portfolio with a longer duration than that of the liability.

Vanguard's view, supported by the rigorous quantitative analysis explained in this paper, is that the most efficient approach to mitigating surplus risk is to deploy the liability-hedging allocation primarily against the longest-dated portion of the pension liability—that is, to use the last of the three approaches listed above.

We furthermore assert that a pension sponsor should be most focused on mitigating a plan's total surplus risk rather than managing liability tracking error, as in the second approach, and that the pension investment industry's focus on the second approach often works counter to that goal.

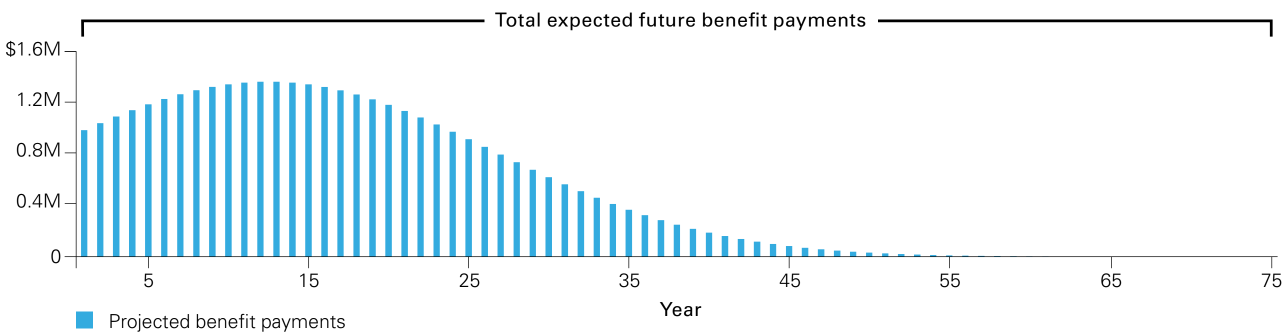
Part 1: Decomposing pension risk by maturity

Pension liabilities are "front-loaded"

Figure 1 shows the year-by-year projected benefit payments for a sample pension plan that we will use throughout this analysis. The sum of the bars is the plan's total expected future benefit payments.[4]

We typically think of a pension liability as a single amount representing the present value of all future expected benefit payments.[5] However, it is equally valid to think of it as the sum of partial liabilities—that is, one distinct liability value for each future year of expected payments.[6]

Figure 1. Projected benefit payments



Source: Vanguard.

4 To limit the scope of this paper, we assume the sample plan is frozen and that future service and/or salary increases need not be considered. For a more in-depth discussion of different liability measurements for plans that are not frozen, see Wolfram and Dutton (2018).

5 The math supporting our results appears in Appendixes B and C. See Appendix A for a primer on the foundations of pension liability measurement.

6 The length of a standard benefit payment projection from an actuarial valuation is often 100 years. Because of our focus on a frozen pension plan, the projected payments in this paper last for 75 years.

As with the liability as a whole, we can analyze each partial liability separately to measure present value, duration, convexity, and even "return" (change in value over a given period of time).

Figure 2 shows the partial liability for each future payment year—that is, the present value of each annual projected payment amount, discounted using corporate bond yields as is typically done for financial reporting purposes. The sum of the bars is the total pension liability.

The total liability can be thought of as a "portfolio" of partial liabilities, with the amount allocated to each based on the present value of each year's projected benefit payments.

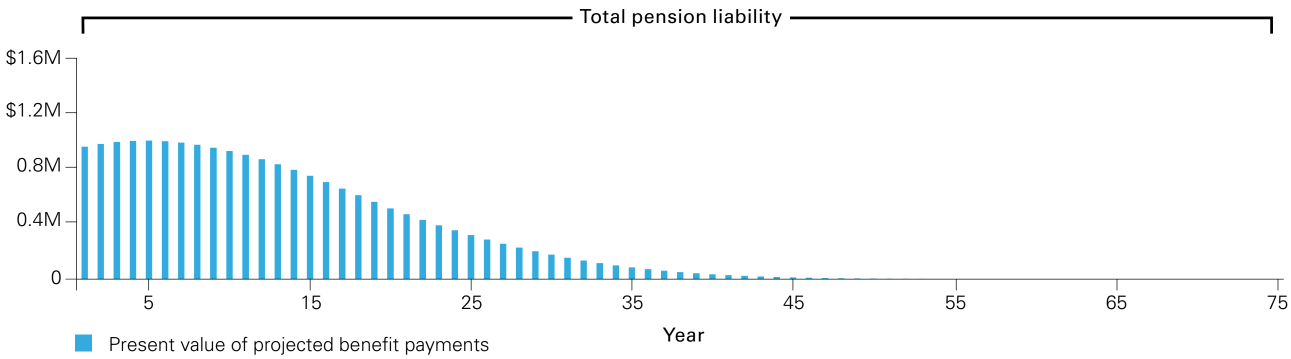
Figure 3 shows the liability allocation for each year as a percentage of the total liability. For instance, the first partial liability (corresponding to the first year of projected benefit payments) represents 4.55% of the overall liability, the second represents 4.65%, and so forth. The sum of the bars is 100% (all of the liability).

As Figure 3 shows, the typical pension liability is front-loaded along the maturity spectrum. In other words, it is primarily attributable to payments due in the early portion of the payment window. More specifically, we find that the first ten years of expected benefit payments account for 46% of the liability, years 11–20 account for 34%, and years 21–30 account for 15%. This leaves only 5% for payments beyond the 30th year. This front-loading is quite typical among pension plans in the United States.

Pension risk is more "back-loaded"

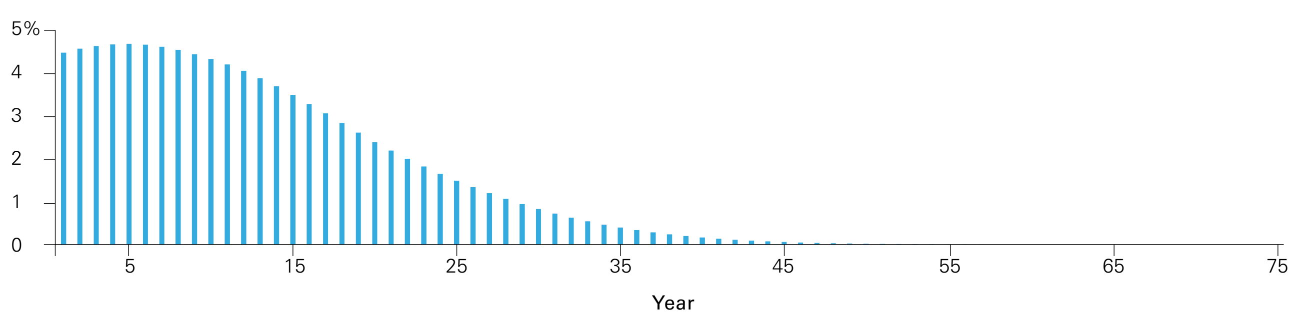
Analyzing each partial liability individually allows us to draw conclusions about the sources of risk along the maturity spectrum. In doing so, we find that liability risk is significantly back-loaded.

Figure 2. Present value of projected benefit payments



Sources: Vanguard, Society of Actuaries.

Figure 3. Allocation of liability to each point in time



Sources: Vanguard, Society of Actuaries.

For this analysis, we calculated returns (changes in value) for each partial liability on a monthly basis using the FTSE Pension Discount Curve from September 1995 to September 2018.[7, 8] This gave us a return series of 276 months.

We performed the following additional calculations to divide the liability risk into individual groups along the maturity spectrum:[9]

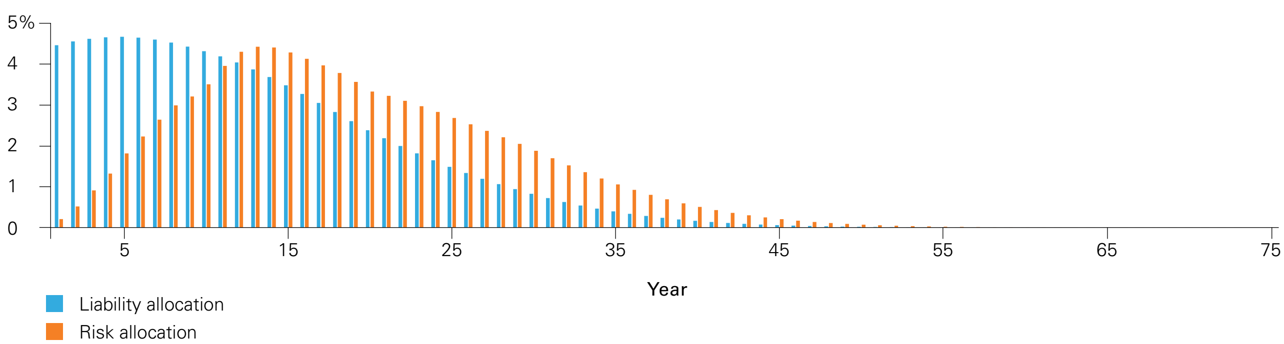
- The standard deviation for each partial liability return series.
- A correlation matrix for the partial liability returns (the correlation of each return series with each other).
- The contribution of each partial liability to the total risk.[10]

Figure 4 shows the results of our analysis. It repeats the information shown in Figure 3 (year-by-year liability allocation) and adds the year-by-year liability risk allocation.

Figure 4 indicates that the source of pension liability risk is significantly more back-loaded than that of the pension liability.

Figure 5 presents a detailed comparison of key drivers of liability value and risk along the maturity spectrum.

Figure 4. Comparison of a liability allocation (front-loaded) to risk allocation (back-loaded)



Sources: Vanguard, Society of Actuaries.

Figure 5. A comparison of liability value and liability risk across maturities

Years	Percentage of liability present value	Percentage of liability risk	Ratio of risk to present value
1–10	46%	20%	0.43
11–20	34%	41%	1.21
21–30	15%	26%	1.73
31–100	5%	13%	2.60

Sources: Vanguard, Society of Actuaries.

7 To calculate each monthly return for a partial liability, we used the discount rate at both the beginning and end of that period. For instance, the ten-year partial liability has an associated discount rate of 3.33% as of December 31, 2016 (using the ten-year spot rate from the FTSE Pension Discount Curve), and is ten years from being paid. One month later, that same partial liability is now nine years and eleven months from being paid and has a discount rate of 3.36% (using a weighted average of the nine- and ten-year spot rates from the January 31, 2017, FTSE Pension Discount Curve). Its return can be calculated by dividing its present value at the end of the period by the present value at the beginning. In this case, the return would be –0.04%.

8 See Society of Actuaries (2019).

9 See Appendix B for additional details of these calculations.

10 Because the total liability is the sum of the individual partial liabilities allocated by size, we used the standard multi-asset portfolio variance calculation formula, $\sigma_p^2 = \sum_i \sum_j \omega_i \omega_j \sigma_i \sigma_j \rho_{ij}$, where i and j are the partial liabilities, σ is their standard deviation, ω is their weights, and ρ is the correlation between their returns. To analyze nearly 100 partial liabilities, this formula was quite long and was solved using matrix multiplication (for details, see Appendix B). If we only had three partial liabilities (A, B, and C), for example, the formula would be the following: $\sigma_p^2 = \omega_A^2\sigma_A^2 + \omega_B^2\sigma_B^2 + \omega_C^2\sigma_C^2 + 2\omega_A\omega_B\sigma_A\sigma_B\rho_{AB} + 2\omega_A\omega_C\sigma_A\sigma_C\rho_{AC} + 2\omega_B\omega_C\sigma_B\sigma_C\rho_{BC}$.

It is again clear that liability risk is significantly back-loaded and weighted toward payments far into the future even when the liability itself is predominantly weighted toward near-term payments. This is because of the interest rate sensitivity (as often measured by duration) of the long-dated payments. Though the present value of those payments is small, their sensitivity makes them important contributors to overall liability risk.

How does this analysis affect Vanguard's approach to constructing liability-hedging portfolios for pension clients? We think the impact is quite significant. When a plan sponsor's liability-hedging assets are less than the total amount of liability, as is typically the case, we believe that concentrating on the main source of risk—that is, the longest-dated future benefit payments—can most efficiently reduce surplus risk.

This implies that, from the perspective of reducing surplus risk, the optimal duration of a pension plan's liability-hedging asset portfolio is often longer than that of the liabilities. In dollar terms, though, the liability-hedging portfolio will not be more sensitive to interest rate changes than the liability will because of its smaller size.

As an example, consider probably the most common practice. A sponsor with a liability-hedging portfolio equal to 40% of the liability value that targets the duration and liability key rate profile in an attempt to hedge all liability equally (strategy 2 above) would hedge 40% of its liability risk. Alternatively, a sponsor could choose to hedge the longest 40% of liability, associated with years 13 through 30 (realizing that few if any bonds are available to hedge liability beyond the 30th year). This strategy would hedge about 60% of the liability risk, making the portfolio more than 1.5x as effective.

Part 2: Decomposing pension risk by source of discount rate movements

We can also decompose returns and attribute risk based on the three key drivers of movements in discount rates, as shown in **Figure 6**.

By attributing monthly liability returns to each of the three factors, we estimate that the contribution to risk is 64% from interest rate changes, 27% from credit spread changes, and 9% from changes in yield curve shape. The math involved is similar to the process described earlier.

Figure 6. Decomposition of pension liability risk by yield curve movement

Factor	Defined as:	Impact on liability return measured as:	Risk attribution
Interest rate changes	Parallel shifts in the U.S. Treasury yield curve	Changes in liability value explained by changes in the 10-year zero-coupon Treasury yield[11]	64%
Credit spread changes	Parallel shifts in the credit spread component of the corporate bond yield curve	Changes in liability value explained by changes in the 10-year point on the FTSE Pension Discount Curve, net of the estimated impact of interest rates	27%
Changes in yield curve shape[12]	Nonparallel shifts in the corporate bond yield curve (flattening, steepening, etc.)	Total change in liability value net of the estimated impacts of interest rates and credit spreads	9%

Sources: Vanguard, U.S. Department of the Treasury.

11 The 10-year Treasury yield is used because it is commonly issued and approximately equal to liability duration.

12 Movements related to shape changes could be further decomposed into changes in the Treasury curve and the corporate curve (one could flatten while the other could steepen). We have combined these for this analysis.

Again, this result has an important practical application. It points to where a pension sponsor's main priorities should lie when constructing a liability-hedging portfolio. In our view, the order of importance is as follows.

First, investing the liability-hedging portfolio with an optimal interest rate sensitivity (duration) will protect against parallel movements in interest rates, thereby accounting for at least 64% of liability risk. The point is that constructing the portfolio with an appropriate duration even without regard to credit spread or yield curve risk can significantly reduce a plan's overall surplus risk.

Theoretically, this type of protection could be accomplished using a liability-hedging portfolio consisting only of Treasury STRIPS with a maturity equal to that of the liability duration. However, this is rarely done in practice. As a practical alternative, investing in a combination of Treasury STRIPS and Treasury bonds with an optimal duration would likely account for more than 64% of liability risk. Because the Treasury bonds have both coupon and principal payments, their payments along the maturity spectrum would also begin to mitigate the risk of yield curve shape change.

Additionally, investing in corporate bonds with optimized duration and credit quality to protect against changes in credit spreads can mitigate an additional 27% of liability risk. A corporate bond can be thought of as having exposure to both the interest rate risk of a Treasury bond and the credit spread risk of the corporate bond. Therefore, corporate bonds with an appropriate duration and reasonably high credit quality (even without regard to key rate duration match) can hedge both interest rate and credit spread risk, or approximately 91% of overall liability risk.[13]

Finally, investing in a range of bonds (both corporate and Treasury) across the yield curve to hedge the liability's key rate durations and protect against changes in the shape of the yield curve will account for the final 9% of liability risk. (Remember that hedging 100% is only possible in theory.) Some pension investors place a high priority on fine-tuning yield curve exposures through derivative overlay strategies. However, this analysis supports the notion that such a degree of precision (and the potential cost and complexity involved) may not materially reduce surplus risk for most pension sponsors.

Conclusion

A typical plan sponsor constructs a liability-hedging portfolio using fewer assets than the plan's total financial reporting liability. This is because plans are generally underfunded and tend to include an allocation to return-seeking assets such as equities.

Under these circumstances, plan sponsors should aim to use liability-hedging assets in the most efficient manner. They should focus on the portions of the liability that contribute the most to risk rather than hedging all components equally.

As shown in this analysis, hedging long-dated liabilities is expected to mitigate surplus risk to the greatest extent possible. This could mean having a duration of liability-hedging assets that is materially longer than that of the liabilities.

Our analysis indicates further that liability-hedging strategies should first concentrate on mitigating the risks from parallel shifts in interest rates and credit spreads, which account for over 90% of total liability risk. Mitigating risk related to changes in the shape of the yield curve by precisely matching the key rate durations of the assets and liabilities could produce marginal incremental benefits but may not be worth the cost and complexity.

References

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Wolfram, Joseph M., and Brett B. Dutton, 2018. *Pension Derisking: Start With the End in Mind.* Valley Forge, Pa.: The Vanguard Group.

13 Because there is no market for corporate STRIPS or for investing in only the corporate spread component of a corporate bond, coupon-bearing corporate bonds must be used. As with coupon-bearing Treasury bonds, this gives the holder some exposure across the yield curve (through various coupons and principal maturities) and helps mitigate the risk of yield curve shape change. Furthermore, although beyond the scope of this paper, since equity maintains some relationship or correlation to the credit spread portion of the corporate bond return, a plan might opt to balance its allocation between corporate bonds and Treasury bonds.

Appendix A. A primer on pension liability valuation and risk measurement

The pension liability is a plan's true investment benchmark

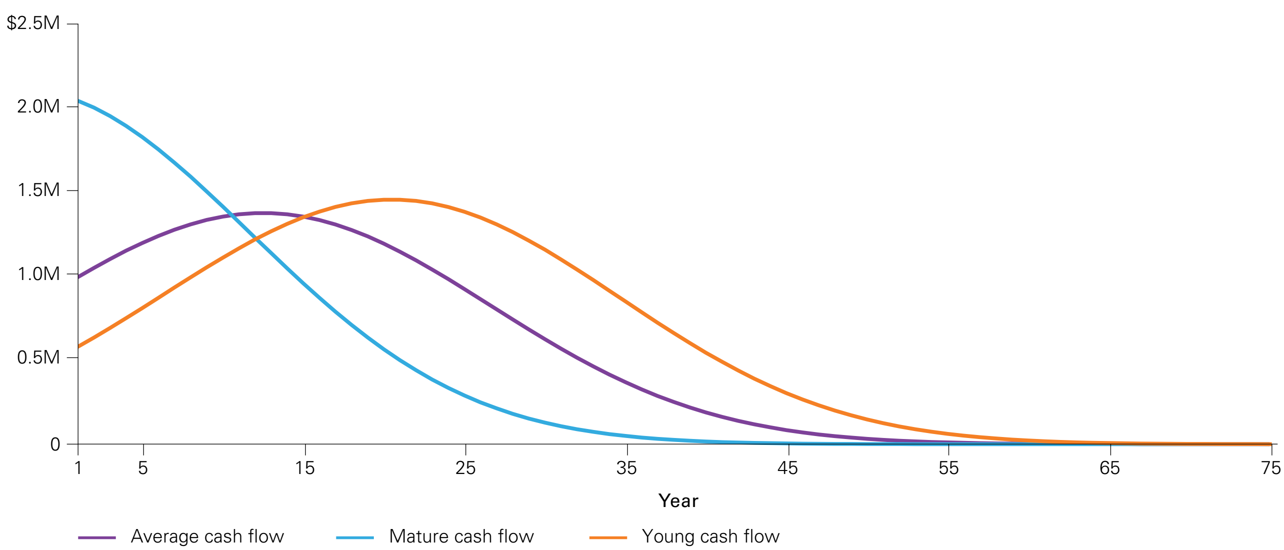
For traditional annuity-based corporate defined benefit (DB) pension plans, the liability is valued by creating a set of expected benefit payments (cash flows).[14] A present value is then placed on each of those cash flows using the discount factors derived from a high-quality corporate bond spot-yield curve.

A cash-flow pattern may look like any of those shown in **Figure A-1**, labeled "young," "average," and "mature" based on the demographics of the plan's participants. A young plan will have more active participants than retirees, with cash flows peaking at a year far in the future. A mature plan will have more retirees and may have already had its peak cash-flow year.[15]

The period-to-period movement of the present value of these cash flows is known as the liability return. This becomes the true benchmark for asset performance and, more specifically, for the liability-hedging portion of the investment portfolio. By "true benchmark," we mean that the asset growth must keep pace with the liability growth or the company will need to make annual contributions to fill that gap. Also, asset risk should be aligned with liability risk to avoid unfunded gaps during times of market stress.

To have a successful LDI program, plan sponsors must understand the risk of the liability and the major drivers of that risk. They can then identify an investment strategy that most efficiently mitigates the overall risk of the plan's key asset/liability measures, such as surplus or funding ratio.

Figure A-1. Various cash-flow patterns ranging from young to average to mature



Notes: Each set of cash flows is designed to have the same liability amount but different demographics. This figure is for illustrative purposes only. It is not based on any particular portfolio.

Source: Vanguard.

14 The plan's actuary creates a projection of expected benefit payments during the annual actuarial valuation process. This involves using the plan's benefit formula, each participant's service and compensation data, and the plan's demographic assumptions (retirement, termination, disability, and death). These combine to determine the amount and timing of the expected benefit payments (often, and throughout this paper, called cash flows).

15 In terms of plan design, a young plan most likely is open to new participants and has ongoing benefit accruals. A mature plan would be closed to new participants and frozen to future benefit accruals.

The pension liability is "bond-like"

The discount rate curve used for valuing corporate pension liabilities is typically constructed from high-quality corporate bonds, AA-rated for accounting liability and A- to AAA-rated for IRS funding regulations. A discount rate curve, as shown in **Figure A-2**, is a series of spot yields that can act as discount rates to determine the present value of future cash flows. For instance, the cash flow payable in one year is discounted for one year at the one-year spot rate. The pension liability is the sum of the present value of these individual future cash flows.

The liability is often described as "like a bond," specifically, a corporate bond or a combination of a treasury bond and the spread component of a corporate bond, for two reasons. The cash flows are generally fixed into the future,[16] much like the coupon and principal payments of a bond. And they are valued using a corporate discount rate curve, which makes them subject to many of the same risks as bonds, specifically interest rate risk and credit spread risk.[17] Further, a pension liability has many of the same financial relationships as bonds, such as a value with an inverse relationship to yields: the higher the yield, the lower the liability, and vice versa.

Measuring liability risk

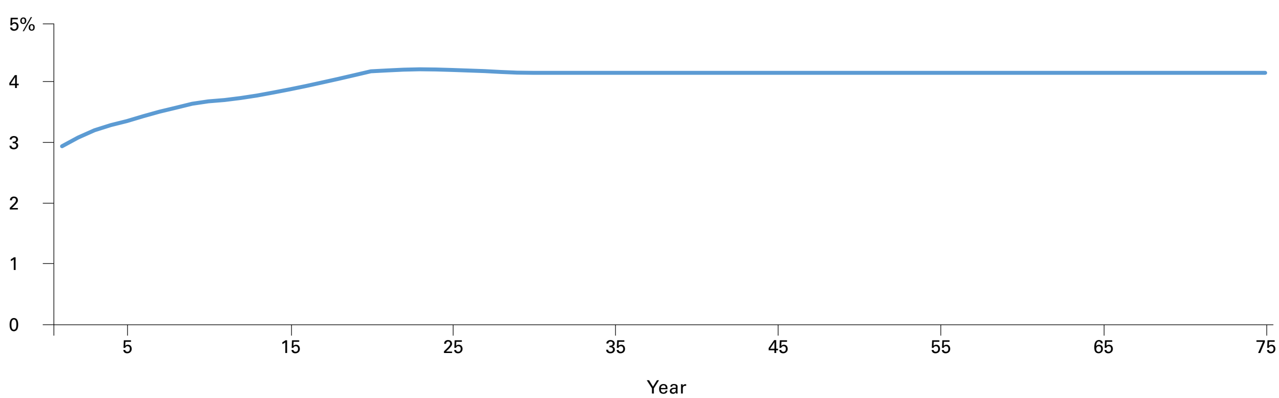
The risks associated with a pension liability are commonly measured and summarized in the same way as bond risks. These measures include the following.

Duration: Measures the sensitivity of liabilities (or bond price) to interest rate movements. The higher the duration, the more sensitive the liabilities. Standard duration measures usually assume parallel changes in interest rates and a linear relationship between interest rate change and price change. This may not hold for large interest rate movements.[18]

Convexity: Measures the nonlinear relationship of liabilities (or bond prices) to changes in interest rates, or the second derivative of the value of liabilities to interest rates.[19]

Dollar value of a basis point (DV01): The change in value of a liability given a one-basis-point change in the yield curve.

Figure A-2. A sample yield curve for discounting pension cash flows to determine liabilities



Notes: This figure is for illustrative purposes only. It is not based on any particular portfolio.
Source: Society of Actuaries.

16 With some variation based on demographic experience.

17 Though not default risk, as benefit payments are guaranteed by the sponsor and backed by the PBGC.

18 Estimated liability (duration only) = initial liability x (1 – duration x discount rate change), a good estimate of the change in liability.

19 Estimated liability (duration and convexity) = initial liability x (1 – duration x discount rate change + ½ x convexity x discount rate change ^ 2), a better estimate of the change in liability.

Figure A-3 shows these risk measures for our three different cash-flow patterns.

Key rate duration: An attribution of the duration to various maturities, this helps measure risk associated with a change in the shape of the yield curve (flattening or steepening), as shown in **Figure A-4**.

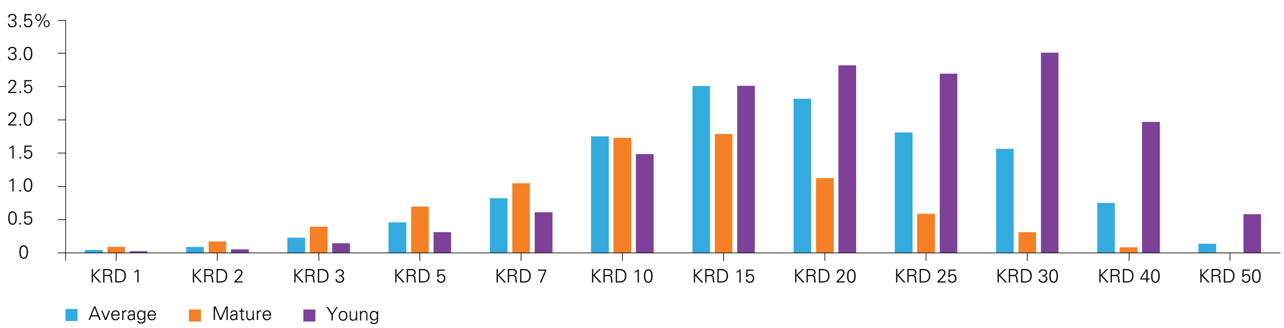
Key rate DV01: As shown in **Figure A-5**, this is similar to key rate duration. An attribution of the overall DV01 to various maturities, it also helps measure the risk associated with a change in the shape of the yield curve.

Figure A-3. Liability statistics for various duration cash flows

						Ten-basis-point increase			
Cash flow	Duration	Convexity	DV01	Initial liability	Liability with ten-basis-point increase in discount rates	Estimated liability (duration only)	$ difference	Estimated liability (duration and convexity)	$ difference
Average	12.48	242.06	26,583	21,318,329	21,054,797	21,052,38	(2,559)	21,054,818	21
Mature	8.03	110.27	17,102	21,318,329	21,148,359	21,147,191	(1,168)	21,148,366	7
Young	16.21	381.67	34,526	21,318,329	20,976,691	20,972,661	(4,029)	20,976,730	39

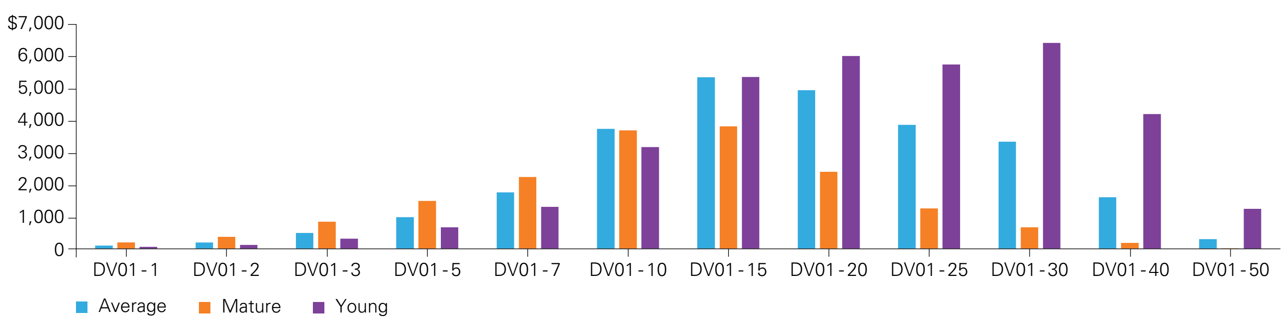
Sources: Vanguard, Society of Actuaries.

Figure A-4. Liability statistics for various duration cash flows based on key rate durations



Sources: Vanguard, Society of Actuaries.

Figure A-5. Liability statistics for various duration cash flows based on DV01



Sources: Vanguard, Society of Actuaries.

Appendix B. Mathematical details of our analysis—risk allocation along the yield curve

This section outlines the mathematics behind our method of attributing liability risk to various projected benefit payments or partial liabilities. To simplify our example, we have abbreviated the math by grouping the expected benefit payments to occur at only 12 specific years, the key rates, as opposed to 100 years. The key rates we have used are Years 1, 2, 3, 5, 7, 10, 15, 20, 25, 30, 40, and 50.

Figure B-1 shows the results of the first part of our analysis. Terms used include the following.

Year (i): The year in which the expected benefit payments are paid.

Present value cash flow—PVCF(i) or L(i): The present value of the expected benefit payments for each year, referred to as "partial liabilities" in this paper. The sum of these will equal the total liability. All liability has been attributed to a key rate; it would be the same if we used all 100 years.

Liability allocation—LA(i) = L(i)/L: The percent of the total that each partial liability represents. The sum of these equals 100%. Regarding the total liability as a portfolio of liabilities, we can say, for example, that 4.55% of the overall liability is allocated to partial liability at Year 1.

To construct the figure, we developed a return series for each of the partial liabilities using the liability allocation and monthly high-quality corporate yield curves from September 1995 to September 2018. The return during a given month is the rate of change in the liability from the beginning to the end of the month, using the discount rates from the respective yield curves.

Standard deviation for each partial liability: This is the standard deviation of the return series.

Correlation matrix: These are the correlations of each partial liability's return series with each other.

We then calculated the variance and standard deviation of the overall liability using the multi-asset portfolio variance formula

$$\sigma_\rho^2 = \sum_i \sum_j \omega_i \omega_j \sigma_i \sigma_j \rho_{ij}$$

where i and j are the partial liabilities, σ is their standard deviation, ω is their weights, and ρ is the correlation between their returns.

Figure B-1. Standard deviation and correlation matrix for various partial liabilities based on key rates

Liability(L)= sum of L(i)	21,318,329			Correlation matrix											
Year, (i)	Present value cash flow, PVCF(i) or L(i)	Liability allocation, LA(i) = L(i)/L	Standard deviation for each partial liability	1	2	3	5	7	10	15	20	25	30	40	50
1	970,380	4.55%	0.31%	1.0000	0.9096	0.8093	0.6790	0.6058	0.5093	0.4212	0.2848	0.2898	0.2775	0.2808	0.2848
2	990,651	4.65%	0.55%	0.9096	1.0000	0.9702	0.8719	0.8019	0.7009	0.5922	0.4470	0.4235	0.3868	0.3885	0.3903
3	1,510,792	7.09%	0.83%	0.8093	0.9702	1.0000	0.9502	0.8952	0.8019	0.6916	0.5404	0.5144	0.4703	0.4713	0.4723
5	2,027,010	9.51%	1.42%	0.6790	0.8719	0.9502	1.0000	0.9765	0.9009	0.8094	0.6355	0.6246	0.5860	0.5877	0.5890
7	2,483,309	11.65%	1.95%	0.6058	0.8019	0.8952	0.9765	1.0000	0.9555	0.8843	0.6955	0.6990	0.6639	0.6659	0.6675
10	3,662,160	17.18%	2.58%	0.5093	0.7009	0.8019	0.9009	0.9555	1.0000	0.9453	0.7743	0.7931	0.7486	0.7497	0.7503
15	3,763,190	17.65%	3.75%	0.4212	0.5922	0.6916	0.8094	0.8843	0.9453	1.0000	0.9025	0.8895	0.8026	0.8040	0.8047
20	2,618,268	12.28%	4.54%	0.2848	0.4470	0.5404	0.6355	0.6955	0.7743	0.9025	1.0000	0.9283	0.7793	0.7761	0.7722
25	1,636,726	7.68%	5.66%	0.2898	0.4235	0.5144	0.6246	0.6990	0.7931	0.8895	0.9283	1.0000	0.9427	0.9405	0.9375
30	1,141,847	5.36%	7.59%	0.2775	0.3868	0.4703	0.5860	0.6639	0.7486	0.8026	0.7793	0.9427	1.0000	0.9996	0.9984
40	448,814	2.11%	10.22%	0.2808	0.3885	0.4713	0.5877	0.6659	0.7497	0.8040	0.7761	0.9405	0.9996	1.0000	0.9996
50	65,181	0.31%	12.92%	0.2848	0.3903	0.4723	0.5890	0.6675	0.7503	0.8047	0.7722	0.9375	0.9984	0.9996	1.0000

Sources: Vanguard, Society of Actuaries.

If we were to expand this, we would have 144 results for our abbreviated 12-partial-liability calculation (12 partial liabilities x 12 partial liabilities).[20] The first 12 rows are shown in **Figure B-2** and are associated with the partial liability for Year 1.

Summing the last column (across all 144 rows, not just the ones shown here), the monthly liability variance is 0.0869% and the standard deviation (the square root of the variance) is 2.9487%.[21]

Figure B-2. Calculation of liability variance using key rate durations

Asset (i)	Asset (j)	(1) weight or ω(i)	(2) weight or ω(j)	(3) Standard deviation or σ(i)	(4) Standard deviation or σ(j)	(5) Correlation or ρ(i,j)	Total liability variance 0.0869% Product = (1) * (2) * (3) * (4) * (5)
1	1	4.552%	4.552%	0.311%	0.311%	1.0000	0.000002%
1	2	4.552%	4.647%	0.311%	0.553%	0.9096	0.000003%
1	3	4.552%	7.087%	0.311%	0.828%	0.8093	0.000007%
1	5	4.552%	9.508%	0.311%	1.418%	0.6790	0.000013%
1	7	4.552%	11.649%	0.311%	1.946%	0.6058	0.000019%
1	10	4.552%	17.178%	0.311%	2.584%	0.5093	0.000032%
1	15	4.552%	17.652%	0.311%	3.749%	0.4212	0.000040%
1	20	4.552%	12.282%	0.311%	4.545%	0.2848	0.000023%
1	25	4.552%	7.678%	0.311%	5.665%	0.2898	0.000018%
1	30	4.552%	5.356%	0.311%	7.592%	0.2775	0.000016%
1	40	4.552%	2.105%	0.311%	10.218%	0.2808	0.000009%
1	50	4.552%	0.306%	0.311%	12.917%	0.2848	0.000002%
Variance of partial liability 1, L(1):							0.00018%

Sources: Vanguard, Society of Actuaries.

20 Note that the 100-partial-liability version would have 100 x 100, or 10,000, terms.

21 When the math is done for all 100 partial liabilities, the liability variance is actually 0.0886% and the standard deviation 2.9772%.

Next, we determined the variance for each partial liability and divided that by the variance of the total liability to attribute risk to each partial liability.

For this step, we summed the partial liability variance using the same formula referred to above but did separate calculations for i = 1, i = 2, i = 3, and so forth. As an example, to calculate the variance for partial liability 1 we added the last column from Figure B-2. The results are shown in **Figure B-3**.

The sum of partial variance across all 12 partial liabilities equals the total liability variance. Therefore, we can divide each partial variance by the total variance for an attribution of risk from each partial liability.

As shown in Figure B-3, 0.210% of the risk is attributable to the first partial liability (the projected benefit payment at Time 1). This is a function of both the size of the payment (relative to the total) and the volatility of its value (which is dependent on the volatility of interest rates and the duration of the payment).

Notice that the attribution starts off very low for early partial liabilities. This is because even though the yield curve might be volatile at that point and the allocation to the liability high, the sensitivity of that liability, as shown by the standard deviation, is low (it has a low duration). The attribution tends to increase with time even though the liability allocation may stay the same or even fall. Thus, the allocation to the third- and the 25th-year partial liabilities are about the same—7.09% and 7.68%—but the risk allocations are 1.401% and 14.027%. The 25th-year partial liability contributes 10x as much risk as the third-year, even though they have the same present value allocation.

Another comparison can be made between the seventh- and the 40th-year partial liabilities. Each has a risk allocation of about 6.5%, even though the present value of the seventh-year partial liability is more than five times higher. This is because the sensitivity of the partial liability increases with time (see the standard deviation column). Eventually, this sensitivity overwhelms the liability's allocation in terms of its contribution to risk. This is an additional example of liability present value being front-loaded and risk allocation back-loaded.

Figure B-3. Attribution of risk across key rate durations

Year (i)	Liability allocation, LA(i) = L(i)/L	Standard deviation for each partial liability	Partial variance of L(i)	Risk attribution = variance of L(i) / total variance L
1	4.55%	0.31%	0.00018%	0.210%
2	4.65%	0.55%	0.00046%	0.528%
3	7.09%	0.83%	0.00122%	1.401%
5	9.51%	1.42%	0.00323%	3.713%
7	11.65%	1.95%	0.00585%	6.728%
10	17.18%	2.58%	0.01222%	14.050%
15	17.65%	3.75%	0.01902%	21.880%
20	12.28%	4.54%	0.01511%	17.384%
25	7.68%	5.66%	0.01220%	14.027%
30	5.36%	7.59%	0.01074%	12.350%
40	2.11%	10.22%	0.00568%	6.532%
50	0.31%	12.92%	0.00104%	1.198%
Total liability variance			0.0869%	

Sources: Vanguard, Society of Actuaries.

Figure B-4. Comparison of present value and risk for groups of partial liabilities

Partial liabilities	Present-value allocation	Risk allocation	Risk per present value
KRD 1, 2, and 3	16.3%	2.1%	0.13
KRD 5, 7, and 10	38.3%	24.5%	0.64
KRD 15, 20, and 25	37.6%	53.3%	1.42
KRD 30, 40, and 50	7.8%	20.1%	2.59

Sources: Vanguard, Society of Actuaries.

Figure B-4 shows another way to look at this, by dividing the partial liabilities into four groups.

Appendix C. Mathematical details of our analysis—risk allocation based on drivers of corporate discount yield curve changes

This section shows the mathematics behind our attribution of liability risk to drivers of yield curve changes. The process is similar to that shown in Appendix B. But it is simpler because we use only three drivers of yield curve change, as opposed to 12 projected benefit payments (and 100 in the paper itself). The drivers were summarized in Figure 6.

Figure C-1 shows the results of the first part of our analysis. Terms used include the following.

Key driver. The three main sources of movement in the corporate discount curve.

Allocation. Because the yield curve (and therefore return) is affected each month by all three drivers, each receives a 100% allocation to the portfolio.

To construct the figure, we developed a return series for each of the key drivers based on the yield curves (both corporate and Treasury) used to determine allocation and the monthly high-quality corporate yield curves from September 1995 to September 2018. The return during a given month is the rate of change in the liability from the beginning to the end of the month, using the yield curves' discount rates.

Standard deviation for each driver. The standard deviation of the above-mentioned return series.

Correlation matrix. The correlations of each key driver's return series with each other.

We then calculated the variance of the overall liability and the standard deviation of that liability, again using the multi-asset portfolio variance formula

$$\sigma_\rho^2 = \sum_i \sum_j \omega_i \omega_j \sigma_i \sigma_j \rho_{ij}$$

where i and j are the drivers, σ is their standard deviation, ω is their weights (100% in all cases), and ρ is the correlation between their returns.

Figure C-1. Standard deviation and correlation matrix for drivers of yield curve changes

Key driver	Allocation	Standard deviation of associated return series	Interest rates	Credit spreads	Yield curve shape
Interest rate changes	100%	3.148%	1.0000	–0.2227	–0.3775
Credit spread changes	100%	2.057%	–0.2227	1.0000	–0.1982
Changes in yield curve shape	100%	1.204%	–0.3775	–0.1982	1.0000

Sources: Vanguard, U.S. Department of the Treasury.

Expanding this out provides 9 (3 x 3) results, as shown in **Figure C-2**.

Summing the last column, the monthly liability variance is 0.0886%. The monthly liability standard deviation (arrived at by taking the square root of the variance) is 2.9772%.

Next, we determined the variance for each driver and divided that by the variance of the total liability.

For this step, we summed each driver's partial liability variance using the same formula as above, but we did the math separately for i = 1 (interest rates), i = 2 (credit spreads), and i = 3 (yield curve changes).

In essence, for the interest rate driver we added the last column from the first three rows, for the credit spread driver we added the last column from rows 4 to 6, and so forth. The results are shown in **Figure C-3**.

The sum of partial variance across all three partial liabilities less the impact of correlation between them equals the total liability variance. Therefore, we can divide each partial variance by the total to determine each partial liability.

As shown in the figure, 64% of the risk is attributable to Treasury interest rate risk, 27% to credit spread risk, and the remaining 9% to yield curve risk.

Figure C-2. Calculation of variance based on drivers of yield curve changes

							Total liability variance 0.0886%
		(1)	(2)	(3)	(4)	(5)	
Asset (i)	Asset (j)	weight or ω(i)	weight or ω(j)	Standard deviation or σ(i)	Standard deviation or σ(j)	Correlation or ρ(i,j)	Product = (1) * (2) * (3) * (4) * (5)
Interest rates	Interest rates	100%	100%	3.148%	3.148%	1.0000	0.099117%
Interest rates	Credit spreads	100%	100%	3.148%	2.057%	–0.2227	–0.014418%
Interest rates	Yield curve	100%	100%	3.148%	1.204%	–0.3775	–0.014311%
Credit spreads	Interest rates	100%	100%	2.057%	3.148%	–0.2227	–0.014418%
Credit spreads	Credit spreads	100%	100%	2.057%	2.057%	1.0000	0.042295%
Credit spreads	Yield curve	100%	100%	2.057%	1.204%	–0.1982	–0.004909%
Yield curve	Interest rates	100%	100%	1.204%	3.148%	–0.3775	–0.014311%
Yield curve	Credit spreads	100%	100%	1.204%	2.057%	–0.1982	–0.004909%
Yield curve	Yield curve	100%	100%	1.204%	1.204%	1.0000	0.014503%

Sources: Vanguard, U.S. Department of the Treasury.

Figure C-3. Attribution of risk based on drivers of yield curve changes

Key driver	Allocation	Standard deviation	Partial variance	Risk attribution
Interest rates	100.00%	3.15%	0.0991%	64%
Credit spreads	100.00%	2.06%	0.0423%	27%
Yield curve	100.00%	1.20%	0.0145%	9%
Preliminary liability variance			**0.1559%**	
Impact of correlation			–0.0673%	
Total liability variance			**0.0886%**	

Sources: Vanguard, U.S. Department of the Treasury.

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