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The role of private equity in strategic portfolios

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- Private equity represents a growing opportunity set for investors, with the potential to significantly enhance returns compared with public investments through exposure to liquidity risk premia as well as the potential for alpha through active selection of and assistance to companies that are not accessible in public markets.

- Although private equity shares some common sources of risk and return with public equity, there are key structural differences to consider when deciding whether to include it in a diversified portfolio. Private equity investments are illiquid and require the use of active management, which introduces both illiquidity and manager idiosyncratic (active) risk in the portfolio.

- Conventional asset allocation approaches such as mean-variance efficient frontiers omit liquidity and active risk dimensions of the risk-return trade-off. We introduce a new portfolio construction framework that accounts for private equity's unique risk and return characteristics, and we embed it into our proprietary model, the Vanguard Asset Allocation Model* (VAAM).

- Portfolio analyses and simulations based on VAAM demonstrate that private equity can play a significant role in strategic, long-term, diversified portfolios. However, there is no single recommended allocation for all investors. Private equity allocations depend on each investor's specific set of circumstances, such as the degree of risk tolerance and the ability to find and access high-quality managers.

* Patent pending.

The expansion of the private equity market over the last two decades, coupled with its potential to enhance long-term portfolio returns, has led to increased investor interest worldwide. Investors that utilize private equity believe that its portfolio benefits outweigh the unique set of challenges that are not present in publicly traded liquid assets. The three main issues are:

- Complexity in the structure and mechanics of private equity that lead to unique sources of risk and return versus public investments.
- Data limitations due to lack of standardized publicly available marked-to-market performance reporting.
- Lack of portfolio construction frameworks that can appropriately account for private equity's unique characteristics.

Given these challenges, how should long-term investors build multi-asset portfolios that include private equity? In this paper, we provide a rigorous yet intuitive analytical framework to answer this question.

The paper is organized in four sections. We start by explaining the basic mechanics of private equity investments and offer a broad view of a dynamic and growing private equity universe. In the second section, we explore the key drivers of risk and return and explain how they differ from those of publicly traded equity. Next, we outline statistical approaches to address the significant data limitation issues with private equity performance attribution, and we detail our preferred method. Finally, we show that conventional portfolio construction techniques are insufficient to account for private equity's unique characteristics. We introduce a new portfolio construction framework for private equity, and we embed it into our proprietary model, the Vanguard Asset Allocation Model (VAAM). VAAM is a portfolio construction tool that simultaneously optimizes across systematic and idiosyncratic ("active") risks and returns (see Aliaga-Díaz et al., 2019).

A primer on private equity

Private equity refers to ownership rights in companies with shares that are not listed on public exchanges. These investments provide capital to companies when it is not possible or desirable for them to access the public markets or when there are opportunities to purchase public or private enterprises that are seen as undervalued or poorly managed. Transactions usually have to be privately negotiated.[1]

Owning a stake in private companies provides asset owners (hereafter, investors) with economic exposure to a different set of businesses than those available on public exchanges.[2] This can help investors further expand their holdings in certain industries, and in some cases, afford them exposure to industries that do not currently include any publicly traded companies.

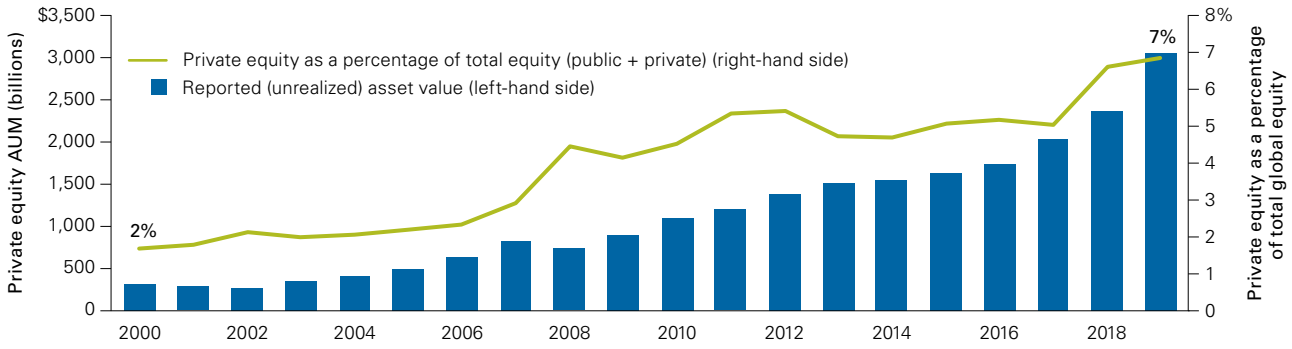
Given the opportunity for investors to potentially earn higher long-term returns, private equity is growing by almost any measure. For instance, based on data from Preqin, the number of private equity funds raised has grown from 634 in 2000 to 1,164 in 2019. Dry powder, which is the amount of capital that is contractually committed to funds but not yet invested ("undrawn"), has grown 9% per year over the same period. According to Pitchbook (2018), the number of private-equity-backed companies in the U.S. grew almost fivefold since the turn of the century, to over 8,000 by the end of 2018. **Figure 1** shows that the asset size of the private equity market has been gradually growing on an absolute basis and relative to the public equity market over the last 20 years.[3] Private equity has risen from 2% to 7% of total investable global equity assets.

1 Exceptions would include selling shares after an initial public offering (IPO).

2 Based on data from Preqin as of September 1, 2019, the largest investor types in private equity were public and private pension plans (40%), banks (14%), insurance companies (13%), asset managers (11%), and sovereign wealth funds (8%).

3 There are a number of potential contributing factors to the maturation of the private equity industry. For example, certain regulatory changes have made it easier for companies to raise private capital, a broader range of investors are willing to deploy capital in private markets, and capital requirements early in a startup's lifecycle have decreased, driven by technological advancements (Ewens and Farre-Mensa, 2020). Significantly more data and theoretical and empirical studies on private equity investments are now available. The growth and maturation of the private segment of the equity market helps make a stronger case for portfolio inclusion.

Figure 1. Private equity continues to grow in absolute and relative terms



Notes: Private equity includes balanced, venture, growth, buyout, turnaround, and secondary funds, as well as co-investment and co-investment multi-manager. The size of the public equity market is proxied by the MSCI All Country World Index.

Sources: Vanguard calculations, based on data from Preqin and Morningstar, Inc.

Access options

Investors can obtain equity exposure to private companies in a few ways.[4] The most common methods are:

- **Directly.** Investors can attempt to buy an ownership stake in a company by directly negotiating with the firm on their own or with syndicate partners. In some cases, investors obtain direct exposure when managers of a private equity fund (hereafter, managers) in which they have a position negotiate the ability to offer them the opportunity to invest directly in a portfolio company alongside the manager's investments through the fund (referred to as a co-investment).
- **Through funds.** Because of the significant challenges in attempting to source and negotiate private equity deals and the special expertise required for robust portfolio company due diligence and consulting, most investors prefer to access private equity through funds. The funds are set up and actively managed by a general partner that pools capital from investors who act as limited partners and deploys the capital in companies that are private or become so as a result of the transaction. For a description of the different styles of funds, see Appendix A.

  Funds are typically closed-ended, and investors have some or full discretion at fund inception as to how much capital they are able to pledge to invest during the life of the fund. Capital is typically committed for an extended period of time, often ten years or longer; contractually required cash contributions (referred to as "capital calls" or "drawdowns") and fund distributions are unspecified in advance, with timing and magnitude controlled by the manager. See Appendix A for more details on the life cycle of a private equity fund.

- **Through funds of funds.** Some investors choose to allocate to a private equity fund of funds, rather than attempting to form a diversified portfolio of private company holdings themselves through direct investing or committing to a group of funds they select. Managers of funds of funds have expertise in fund investing and may have privileged access to certain top-quality managers. Fund of funds managers solicit for capital commitments from investors, and they may invest in private equity funds spanning a number of vintage years.[5] Some also provide liquidity to other private equity fund investors by purchasing their existing interests in funds in a private negotiated transaction, and they accept responsibility for funding any remaining capital commitments and/or directly invest in underlying portfolio companies.

4 It is important to note that private equity is not accessible to all investors. Current owners often have a say when it comes to those interested in direct investments, and managers can have a say in who invests in their funds and often set high minimum investment requirements. In addition, some jurisdictions have laws (such as accredited investor and qualified investor regulations in the U.S.), which can be based on criteria such as total assets, annual income, and level of professional knowledge. Some laws also limit the number of investors per fund.

5 The definition of *vintage year* varies; it can be considered the year when the fund was legally formed, the year when capital was first called, or the year when the first portfolio company investment deal occurred. Spreading investments across funds of different vintage years can help mitigate some liquidity risk, as some cash distributions from older vintage funds can be used to offset capital calls needed for newer funds (Robinson and Sensoy, 2016). This vintage diversification also expands the opportunity set for fund-of-funds managers, as not all direct fund managers raise new funds every year.

Deciding whether to build a portfolio of funds or use a fund of funds

Because there is no investable "index" option to obtain broad-based exposure to the private equity market, investors must be willing to accept some form of active risk. As a result, they may not be comfortable committing their entire private equity allocation to just one manager. In addition, according to a study by Metrick and Yasuda (2010), the median venture capital fund invests in only about 20 companies, and the median buyout fund invests in about half that amount.[6] Similar to findings in Harris et al. (2018), **Figure 2** shows that the cross-sectional dispersion of fund returns has been far wider than that of funds of funds. There is debate about whether investing in a custom set of private equity funds or investing in a fund of funds is the more prudent approach for diversifying across funds. But it does not have to be an either/or decision, and the most appropriate choice can vary by investor for a number of reasons. See Appendix B for details.

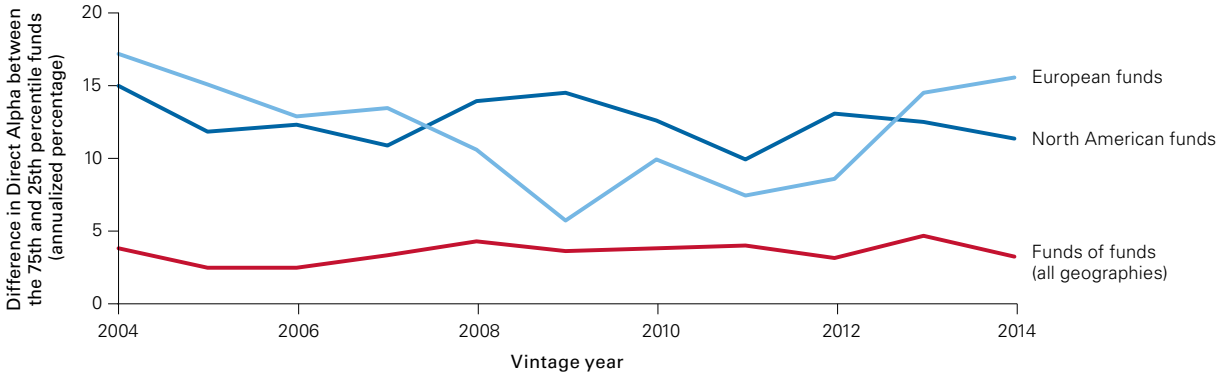
Identifying the drivers of returns for private equity

It is well-accepted by academics and practitioners that the expected return of financial assets should be influenced significantly by their exposure to one or more systematic risks. If investors are accepting a risk that cannot be diversified away, it is rational for them to expect to be compensated for doing so through a higher return. Therefore an important step in determining whether private equity has a role in diversified portfolios is to assess what risks those investors are accepting and how they are similar to or different than risks in their public equity portfolio.

Given that both private and public equity represent ownership stakes in companies that attempt to maximize long-term shareholder value, theory would suggest that they should both share the exposure to systematic market risk. Thus, both public and private equity should earn the equity risk premium, which is the compensation that investors expect for the undiversifiable risk portion of equity investing. However, there are four key reasons why the economic returns of private equity should be different than those of public equity benchmarks:

- **Liquidity premium.** Investors in private equity have less ability to trade their investment and do not control the timing or size of cash flows if invested in funds; therefore, they should require compensation in the form of a liquidity premium.
- **Other risk factors.** The average characteristics of private equity companies may be different than those of public companies (for example, industry, size, financial leverage, geography, and valuation).
- **Manager-specific alpha.** Investors accept idiosyncratic manager-specific risk in exchange for the opportunity to generate alpha.
- **All-in costs.** The size and structure of costs is materially different.

Figure 2. Funds of funds diversify risk, which reduces performance dispersion



Notes: Data reflect all North American and European venture capital funds and buyout funds in the Burgiss Manager Universe database for vintage years 2004–2014. The last evaluated fund vintage year was 2014, because it is a common industry practice to allow most funds to complete their investment period prior to analysis of performance. The Russell 3000 Index is the public market benchmark used for the North American fund sample, the MSCI Europe Index is used for the European fund sample, and the MSCI World Index is used for funds of funds. *Direct Alpha* represents the annualized excess returns of private investments over a public market proxy. It is calculated directly, by discounting each private equity cash flow using an associated public index return factor, rather than indirectly, by first estimating a public market equivalent internal rate of return (IRR) and subtracting this from the private equity IRR. Direct Alpha can be thought of as annualizing the public market equivalent (PME) proposed by Kaplan and Schoar (2005). Returns are net of fund fees. For more information on Direct Alpha, see Gredil, Griffiths, and Stucke (2014).

Sources: Vanguard calculations, using the Burgiss Manager Universe database, as of September 30, 2019.

6 In comparison, the median number of companies held in a U.S. public active equity fund from 2000 to 2017 was 75 (Tidmore et al., 2019).

Liquidity premium

Private equity's higher liquidity risk compared with that of public equities should rationally lead to higher expected returns, with the premium being time-varying, likely higher during economic downturns when liquidity is scarce. This liquidity premium can be attractive for those who have low liquidity constraints; in reality, most long-term investors do not need a 100% liquid portfolio and therefore could handle some exposure to illiquid investments.

There are two different but related forms of liquidity risk: market liquidity and funding (cash flow) liquidity. Market liquidity risk, in general, is the ease with which an investment can be traded. Funding liquidity risk stems from the inflexibility and uncertainty in the size and timing of cash outflows and inflows that a fund investor experiences.

The market illiquidity of private equity exposes investors to at least two potential issues: the inability to liquidate or sell the investment in a timely manner, and a sale in the secondary market at a price potentially well below intrinsic value. Unlike with public equities, if investors want to sell a private equity investment in a secondary market transaction, they must locate a counterparty and negotiate the transaction price. If the investment to be sold is a position in a fund, investors often have to receive permission from the general partner to transfer the ownership to the identified counterparty (Nadauld et al., 2019).

Although the secondary market for public equities is very mature, the secondary market for private equity is shifting from its infancy to adolescence. Even with the growth in the secondary market for private equity stakes, the average fund transaction from 2006 to 2014 was at a 13% discount to net asset value (NAV) (Nadauld et al., 2019).[7] Excluding the outlier year of 2009 (when the average discount to NAV was an astonishing 47%), the average discount still was 9%. Since the global financial crisis, there has been significant growth in secondary market fund transactions, leading to smaller average discounts to NAV. One should still expect secondary prices to fluctuate considerably over the business cycle, consistent with a time-varying liquidity premium that has been exhibited in the cross-section of public equities over time.[8]

The second type of liquidity risk is funding liquidity risk. Fund investors must be flexible enough to make contributions quickly and to deal with potential material delays in distributions from the funds.[9] Both of these liquidity risks can have implications when rebalancing a portfolio with a private equity allocation, something we address in the last section.[10]

Other risk factors

Equity risk factors identified in public equity markets may also be present among private companies. A recent—and significant—body of literature attempts to estimate to what extent private equity may be exposed to some of those common risk factors, such as size and value, and also attempts to determine whether private equity has generated positive net alpha after accounting for those risks.[11]

7 The NAV for private equity funds, unlike that of public equity funds, is typically calculated only once per quarter and is used for reporting purposes only. Each transaction price is privately negotiated between the fund buyer and seller and can be above or below the reported NAV. As these results suggest, historically there has been a supply imbalance, on average, leading to a tendency for the sellers to have to accept a discount to NAV in order to exit their fund position before the end of the fund's full contractual term.

8 Whatever the magnitude of the "market illiquidity penalty" for an investor who is forced to sell, investors who manage liquidity needs appropriately and/or have flexible financial resources can significantly reduce this risk. In fact, some well-capitalized investors and secondary fund managers take advantage of these panic periods by offering to be liquidity providers by purchasing investments at a significant discount to NAV.

9 In order to address this risk, many managers have set up subscription lines that provide a capital call facility for investors. These are loans made to a private fund, secured against investors' committed capital, which lowers both the short-term funding liquidity risk and the administrative burdens by reducing the frequency of capital calls.

10 Robinson and Sensoy (2016) showed that both capital calls and distributions have a procyclical component, with distributions more sensitive than calls during economic downturns. These periods are when investors tend to be most financially constrained, which amplifies their desire to be compensated for accepting funding risk. The authors found that vintage diversification helps reduce funding risk. Additionally, Brunnermeier and Pedersen (2009) found that market liquidity and funding liquidity positively co-vary and reinforce each another.

11 The evidence from the academic literature on alpha and different factor tendencies for private equity varies considerably from study to study, with sample selection, time period, and calculation methodology differences potentially being contributing factors (Korteweg, 2019). Industry representation, geographical allocation, and the idiosyncratic nature of private equity, given that it is a separate sample of companies, can also help explain performance differences between public and private equity. Given the typical level and type of transparency in private equity, some of these differences are hard to measure. As a result of the lack of theoretical or empirical consensus, there is significantly more work to be done to fully understand to what extent the risks and returns of private equity can be explained by factors common in the public markets.

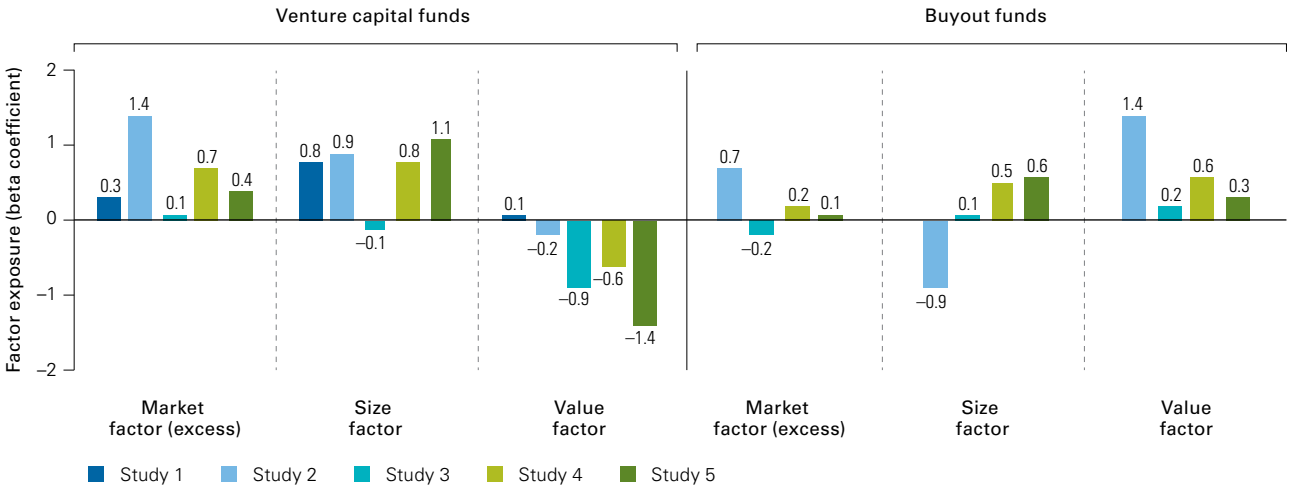
In **Figure 3**, we highlight the market, size, and value factor exposures of the average U.S. private equity fund in a sample of academic studies. The results suggest that these funds tend to have a market beta above one and a significant small size tilt. In addition, buyout funds tend to have a positive value loading, whereas venture capital funds, not surprisingly, tend to have a negative value loading. This evidence is consistent with theory that common risk factors span both public and private companies. Therefore, it is important for investors to determine what risk factor exposures they would like to target or can tolerate from an allocation to private equity.

Manager-specific alpha

Because there is no investable index alternative for the private equity market, investors must accept active risk. The managers of the direct exposure or fund take on the responsibility of trying to prudently add unique value through different actions. We find that managers can attempt to add value in at least five ways:

- **Portfolio company selection.** In addition to their company selection skills, some managers may have access to certain deals or parts of the market that others may not because of their reputation, skill set, or other unique characteristics that the current owners of a private company may find valuable.
- **Thematic bets.** Managers can choose to focus on secular or structural changes (such as technological, regulatory, and consumer preference) that may not be fully reflected in company valuations today.
- **Governance.** Buyout managers typically have full voting control of the company, and they can provide key oversight to help a portfolio company with strategic planning decisions and avoid principal-agent conflicts (e.g., self-dealing) or any other activity that is not aligned with maximizing the company's long-term shareholder value. Where there is not full control of the company, and depending on the level of ownership stake, managers still can take on board of director roles and usually have legal structures where managers receive more control if the company performs poorly.
- **Finance.** Managers may have the talent or ability to source the talent to add value by optimizing the capital structure of portfolio companies.
- **Operations.** Many managers develop sector or industry expertise or focus on certain types of investment situations so they can make key decisions or recommendations. They also can suggest a top consultant or candidates for hire who have significant expertise and skills to help a company cut costs, improve productivity, implement strategic changes, identify attractive acquisition opportunities, and recruit key executives.

Figure 3. Direction and magnitude of risk factor exposures vary by fund style and by study



Notes: Includes five studies of venture capital funds and four studies of buyout funds that utilize the Fama-French (1993) three-factor model to estimate factor exposures of the average private equity fund in their sample. For ease of interpretation, we subtract 1 from the market factor betas.

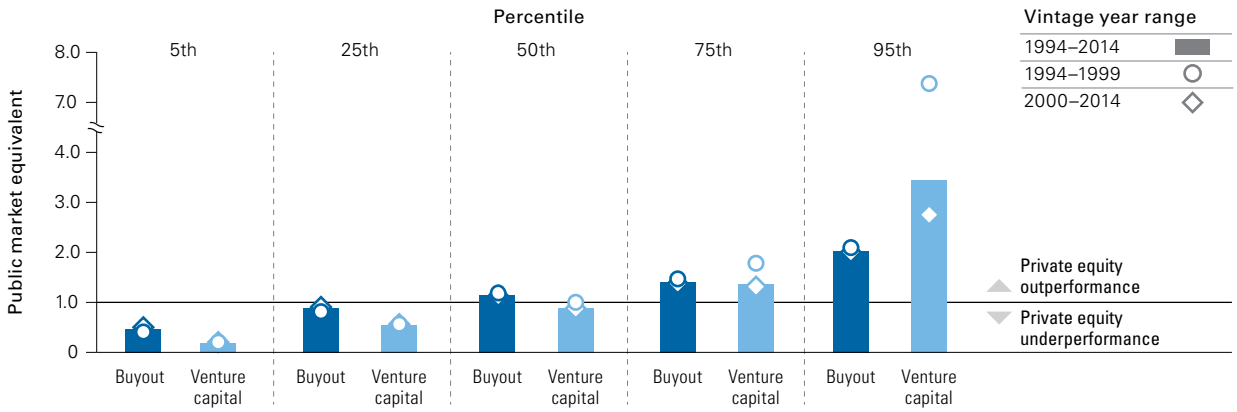
Source: Korteweg (2019).

Although every manager conducts company selection, the extent to which managers apply the other potential ways to add value depends on the unique skills, experience, and network of their firm.[12] Therefore, an investor conducting due diligence should try to understand how a manager attempts to add unique portfolio value, how the manager measures the success of those efforts, and why the manager believes those actions will be successful in the future.

Wallick et al. (2015) find that skillful manager selection is critical given the extreme cross-sectional dispersion of performance among private equity managers—far wider than what has been observed among public active equity funds.[13] Although manager selection criteria are out of scope for this paper, it is important to note that selection involves more than simply evaluating a manager's performance track record (Korteweg and Sorensen, 2017).[14] It can be important to be an early investor with high-quality managers. If the manager's success leads to more demand from investors than the manager can accommodate when raising money for their next fund, those early investors may have a better chance of being selected as limited partners for the new fund, giving them an access advantage.

Venture capital fund performance persistence has remained fairly robust, but these funds have, on average, underperformed public markets since 2000 and with wider dispersion than buyout funds and a positive skew (**Figure 4**). Alternatively, Figure 4 shows that buyout fund returns tended to be higher than public market returns over that same period.[15,16] Taken together, access and selection skill appear to be more critical in venture

Figure 4. Private equity performance has been mixed and varies significantly by fund



Notes: Public market equivalent is calculated using the Kaplan-Schoar (2005) method. The Russell 3000 Index is the public market benchmark used. Returns are net of fees. In line with Harris, Jenkinson, and Kaplan (2016), we start with the 1994 vintage to ensure a meaningful number of funds and compare vintage samples pre- and post-2000. The last evaluated fund vintage year was 2014, because it is a common industry practice to allow most funds to complete their investment period prior to analysis of performance.

Sources: Vanguard calculations, using data from Burgiss.

12 In buyout situations, managers can take into account the expected value-add from these activities when determining how much they are willing to pay for the controlling ownership stake in a company being considered for the portfolio.

13 Because public and private equity have many systematic risks in common, this significant difference in dispersion suggests that the idiosyncratic active decisions that managers make drive a larger percentage of performance variation with private equity funds, which may not be surprising given that each manager tends to invest in a very small subset of companies.

14 David Swensen, the long-time chief investment officer of the Yale University endowment who may be the most well-known evaluator of private equity managers in the world, stresses that qualitative factors (such as people and process) play a central role in manager evaluations (Swensen, 2009). For more information on private equity fund due diligence, see, for instance, Greenwich Roundtable (2010).

15 Since 2000, positive performance persistence among buyout managers has not been as strong as in the past. See Harris et al. (2014), Korteweg and Sorensen (2017), and Braun, Jenkinson, and Stoff (2017) for studies on the evolution of private equity fund performance persistence.

16 Harris et al. (2018) found that funds of funds, on average, have generated net-of-fee returns equal to or above public market indexes. Comparisons of the long-term performance of private and public equity must be considered carefully because of the subjective valuations and fundamentally different cash flow profile with private equity funds. Because private equity fund returns should not be measured using time-weighting given the cash flow control of the manager, the most common direct comparison measure is the PME method proposed by Kaplan and Schoar (2005). It serves as a legitimate opportunity cost measure with an intuitive interpretation by representing how much more or less wealthy an investor would have become by investing in private equity instead of a relevant public equity investment, assuming equivalently timed cash flows in each. For example, if the ratio is 1.2, that means the final wealth is 1.2 times what would have been achieved by hypothetically investing in the chosen public equity index benchmark.

capital.[17] Overall, it is important for investors to set expectations that are specific to the unique set of private equity investments used or being considered and ensure that they conduct skillful manager due diligence.[18]

All-in costs

Investors care most about performance net of all costs. A fairly typical private equity fund fee structure includes a management fee that is paid on committed capital then reduced when the investment period is over, and a performance-based, profit-sharing fee ("carried interest"), which is charged only if the manager generates a net return for the investor that exceeds the contractually agreed-upon hurdle rate (also called *preferred return*).[19]

According to a recent study by MJ Hudson (2020), management fees averaged 1.95% for 2019, with 95% of funds having a step-down after the investment period. From the same study, the vast majority of funds have a carried interest percentage of 20% of profits over the hurdle rate, which they indicate is most often set at an 8% return level. Lastly, some funds have other fees, such as monitoring or transaction fees, which are sometimes fully or partially rebated against management fees.[20]

Investors and fund-of-funds managers who are able to make larger commitments can sometimes negotiate significantly lower fees. The investor must conduct the due diligence necessary to have conviction that the manager can overcome this cost hurdle. As highlighted in Figure 4, many funds have done so over the long term.

Estimating the risk characteristics of private equity

Academics and practitioners alike rely on standard asset-pricing theories to understand the main drivers of risk, both systematic and idiosyncratic, in private equity. This assessment is critical to investors that need to calibrate future expectations for risk and return, as well as covariance with other asset categories, in order to build a prudent portfolio. However, one important challenge with the private equity historical return data is that the limited holdings transparency and reliance on subjective, appraisal-based valuations can lead to significant estimation biases in standard risk factor decomposition models. Thus, applying standard asset-pricing statistical techniques developed for public performance data (which are based on observable, transaction-based marked-to-market valuations) can lead to unreliable and misleading results. As Couts, Gonçalves, and Rossi (2020) state: "The crux of the problem is that we only observe reported (or smoothed) returns, while we need economic (or unsmoothed) returns to evaluate risk exposures and risk-adjusted performance." If investors want to understand the true underlying risk profile of a private investment to fairly compare it with easy-to-measure public assets, they need a valid way to estimate it.

A number of statistical methods have been proposed in the academic literature over the last few decades to try to better understand historical performance.[21] None of them are without shortcomings, which is why there remains no universally agreed-upon approach among academics or practitioners.

17 In unreported results, we find the median PME (using the MSCI Europe Index as the public benchmark) of European buyout and venture capital funds from the 1994–2014 vintage fund sample to be about the same as the style of fund in the U.S. sample.

18 Each investor will have his or her own methods of attempting to identify superior managers and developing alpha and risk factor expectations for them. This is typically best done through a rigorous due-diligence process combined with an understanding of alpha and factor exposure ranges and sensitivity to the probability of success. Otherwise, given the extremely wide cross-sectional distribution of manager performance, the utility of the forecast used to determine the potential role in a portfolio is questionable.

19 Carried interest can help better align the incentives of the manager with the investors.

20 For details on various private equity fund fees, see Robinson and Sensoy (2013) and Phalippou, Rauch, and Umber (2018). The potential tax costs and advantages of private equity can be complicated and vary by investor type, account type, jurisdiction, and the investment's legal structure. Given the dependencies and complexity, providing estimates is challenging. Ultimately, investors considering private equity should ensure that they understand both the tax impacts (if any) to their portfolios and the resources they would need to devote each year to determining and reporting them.

21 There are time-series-based approaches that use both fund cash flows and quarterly NAVs with statistical techniques to obtain economic (unsmoothed) returns that are closer to the hypothetical but unobserved marked-to-market returns. These methods allow for estimations of true factor sensitivity and alpha. Alternatively, there are intricate approaches that use just cash flow data to infer the implied time series of private equity returns. These methods can be complex, requiring, for instance, a Bayesian-based methodology or a generalized method of moments (GMM) approach, and require a number of assumptions. A third method assesses the systematic exposures of private equity funds by analyzing the attributes of portfolio company holdings. This requires a consistent and high level of transparency about holdings as well as judgment in converting into factor sensitivity levels and, as a consequence, alpha estimation. Lastly, building-block methods attempt to simplify the process by breaking down returns into economically reasonable parts, such as growth rate, dividend yield, and risk-free rate. This requires subjective choice of appropriate building blocks.

In this paper, we follow a time-series technique to "unsmooth" reported private equity returns. The use of "smooth" appraisal-based private equity valuations would result in an underestimation of the fund's equity market sensitivity (i.e., market beta) and volatility of returns, thereby leading to overestimation of its risk-adjusted performance. In a portfolio setting, the underestimated volatility could lead to an overallocation to private equity.

We start by sourcing the global, pooled quarterly net returns to limited partners for venture capital and buyout direct funds from the Burgiss Manager Universe from December 31, 2001, to September 30, 2019.[22] The returns are asset-weighted across the private equity fund universe, so the return series represents a reasonable (noninvestable) private equity benchmark. We then apply an econometric approach by using a type of Geltner (1993) method to transform the time series from reported (smoothed) returns to an estimate of marked-to-market (unsmoothed) returns. The process consists of two steps:

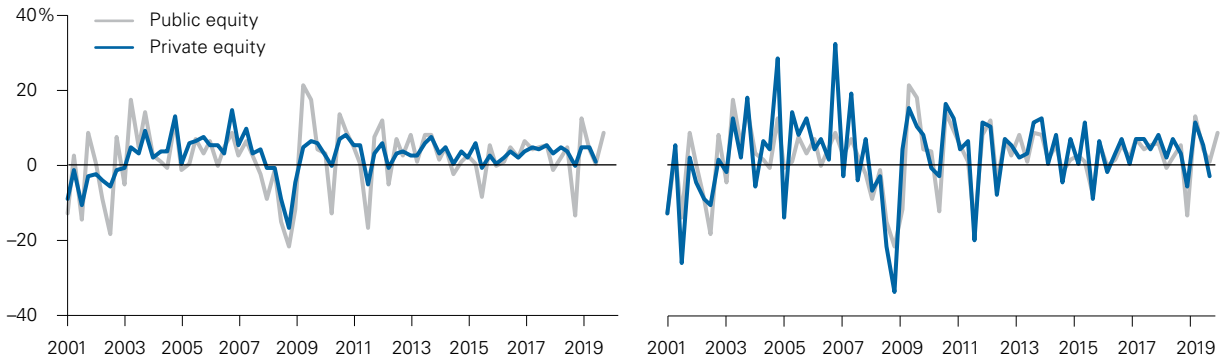
- Estimating the autocorrelation artificially introduced by the appraisal-based reporting and using that estimation to solve for a serially independent version of the return series. The resulting unsmoothed private equity returns should exhibit negligible first-order autocorrelation just as public equity indexes do.
- Calculating the ratio of variances between reported and marked-to-market estimated private equity returns from several studies (for example, Ang et al., 2018) and using that ratio to inflate the variance of private equity returns to be at least 30% higher than those of the public markets.[23]

Thus, the two key assumptions behind this methodology are that true marked-to-market private equity returns should exhibit autocorrelation and volatility levels that are, at a minimum, comparable to the public equity counterparts. As **Figure 5** shows, this approach provides for more economically sensible volatility and covariance estimations while preserving the average return of the private equity investments.

Figure 5. "Unsmoothing" private equity returns results in a more realistic economic return profile

a. As-reported private equity returns, 2001–3Q2019

b. Adjusted private equity returns, 2001–3Q2019



c. Summary statistics

	Public equity	Private equity (as reported)	Private equity (adjusted)
Volatility	16.6%	10.7%	22.6%
Autocorrelation	0.02	0.45	0.00
Annual average return	7.2%	11.0%	11.0%

Notes: The time series of private equity returns is based on global, pooled quarterly net returns to limited partners for venture capital and buyout direct funds from the Burgiss Manager Universe for the period December 31, 2001, to September 30, 2019. The public equity returns are based on the MSCI All Country World Index. The Modified Dietz returns produced by Burgiss are unsmoothed using a Geltner (1993) method, which allows us to remove first-order autocorrelation from the return series to create the adjusted results.

Sources: Vanguard calculations, using Burgiss and Morningstar, Inc.

22 Burgiss utilizes a Modified Dietz calculation methodology as a rough approximation for a time-weighted, appraisal-based return. The chief advantage of the Modified Dietz method is that it weighs each external cash flow by the amount of time it is held in the portfolio without the need to value the portfolio on the date of that external cash flow. We use a start date at the end of 2001, as this represents the period when the private equity fund sample becomes globally balanced and large.

23 A recent survey of investment advisory firms confirms that this variance assumption is reasonable (Horizon Actuarial Services, LLC, 2020).

After applying the unsmoothing procedure to the private equity return data, the next step is to estimate its risk characteristics. Because a private equity benchmark is not investable, the way to estimate private equity's risk attributes is through a risk factor decomposition regression based on publicly traded investable indexes, as follows:

Private equity (uninvestable) benchmark = constant + βx public equity benchmark + δx equity factors premia + ε

Here the "constant" can be interpreted as the excess return of private equity over and above public benchmarks. The volatility of the regression residual, ε, is also important, because it measures a source of factor risk that is specific to private equity, namely the volatility of the illiquidity premium of private equity.

Figure 6 displays the results of this risk factor decomposition. As previously discussed, using the as-reported (appraisal-based) return series for the risk factor regression would lead to downward biases in the market beta exposure and volatility and to an upward bias in the estimated excess return (i.e., the constant term in the regression). The adjusted return column shows unbiased betas and volatilities. We do not find statistically significant loadings for the size or value factors.[24]

Finally, in a full risk assessment of private equity, we also need to account for the active risk component. As discussed in previous sections, private equity can be accessed only by actively investing in a subset of companies directly or through actively managed funds or funds of funds. Thus, in addition to the broad, public systematic risk exposures and the volatility of the liquidity risk premium, private equity investors accept idiosyncratic manager-specific risk. This is effectively a form of active risk, with the potential for alpha being the expected compensation for such risk exposure. Thus, total risk decomposition for a hypothetical private equity fund X would be described by the following equation:

Private equity fund X returns = fund X alpha + private equity (uninvestable) benchmark + η

with the active risk measured by the volatility around the benchmark (i.e., the standard deviation of the residual η). To summarize, there are three distinct sources of risk for any private equity investment:

- Systematic risk (more accurately estimated after the unsmoothing procedure), measured via decomposition of factors present in public markets.
- Illiquidity factor risk that is unique to private equity and not observed in public markets.
- Idiosyncratic or active risk, for the specific manager(s) selected.

Figure 6. Comparing reported and adjusted private equity "benchmark" results

	Private equity (as reported)	Private equity (adjusted)
Constant (annualized)	0.08*	0.04*
Equity market beta	0.5*	1.04*
Factor loadings		
Value (global)	–0.10	–0.21
Size (U.S.)	–0.21	–0.09
Standard error, standard deviation(ε) annualized	6.0%	13.2%
Residual autocorrelation (Durbin-Watson test)	Yes	No

* Statistically significant at the 5% level.

Notes: The time series of private equity returns is based on global, pooled quarterly net returns to limited partners for venture capital and buyout direct funds from the Burgiss Manager Universe for the period December 31, 2001, to September 30, 2019. The Modified Dietz returns produced by Burgiss are unsmoothed using a Geltner (1993) method, which allows us to remove first-order autocorrelation from the return series. Factor proxies used can be found in Appendix C.

Sources: Vanguard calculations, using data from Burgiss and Morningstar, Inc.

24 There are a number of potential reasons for the lack of systematic factor loadings: (1) our private equity data pools venture capital and buyout fund cash flows and quarterly values; thus, some of the specific factor loadings may offset in the aggregate; (2) our small factor proxy is based on the U.S. market only, while our private equity sample is global; and (3) the characteristics of the companies that make up the public factors can be significantly different than those of the private universe. For example, the financial sector tends to be a negligible percentage of the private equity universe but represents a material percentage of public equity.

Private equity return simulations

This risk assessment of private equity is not only insightful on its own, it is also a critical step in the portfolio construction process. In any portfolio framework, a forward-looking estimation of the associated investment risks is as important as the return expectations or excess return targets of the funds selected. In practice, one way to account for both forward-looking risk and return expectations in a portfolio is through stochastic (Monte Carlo) simulation of the attributes of the fund performance.

As a practical use-case application of this framework, here we simulate the returns of a generic private equity fund that has similar risk and return characteristics as our estimation in Figure 6. For simplicity, in this case we do not distinguish variation driven by liquidity risk and idiosyncratic manager risk; instead, we just keep track of the volatility of the combined excess returns.

Figure 7 summarizes the key characteristics of a generic private equity manager we choose to simulate. We generate 10,000 Monte Carlo simulations of the fund returns by combining simulations of the global public equity index produced by our Vanguard Capital Markets Model® (VCMM) with 10,000 independent simulations for the manager's excess return (based on the estimated excess return and its volatility from Figure 7).[25]

Portfolio construction with private equity

How should private equity investors allocate capital among public liquid assets and private illiquid assets in a portfolio? Given some of the requisite features of private equity, namely liquidity and idiosyncratic active risk, what are the implications for standard portfolio construction frameworks? How should they be modified in order to account for the inherent liquidity risks?

In the following sections, we briefly describe our proprietary asset allocation model, the Vanguard Asset Allocation Model (Aliaga-Díaz et al., 2019), and its key properties. We then highlight the features of private equity that make it different from a liquid public asset and show how we approach it from a portfolio modeling standpoint. Finally, continuing with our use-case of a generic private equity manager from the previous section, we present examples of optimized portfolios with private equity allocations, along with sensitivity testing.

The Vanguard Asset Allocation Model (VAAM)

Conventional asset allocation approaches for active-passive blended portfolios have various shortcomings:

- Mean-variance optimization approaches fail to explicitly account for the attributes of active investments (manager idiosyncratic risk, alpha expectations, sensitivity to market beta, and factor loadings) and how they may affect the overall asset allocation.
- Other ad-hoc sequential allocation rules ignore volatilities and correlations among the asset classes, thus failing to correctly assess active-passive-factor risk-return trade-offs. The sequential decisions typically made are, first, determining the allocation to broad asset classes via mean-variance optimization approaches, followed by determining sub-asset allocation on an ad-hoc basis, and finally, determining the active-passive split within sub-asset classes without a robust underlying framework.
- None of the approaches considers an investor's attitude toward alpha (active) risk within a robust framework.

Figure 7. Return and risk attributes of a generic private equity manager

	Return and risk attributes
Total net excess return (estimated liquidity premium + specific-manager alpha)	4%
Equity market beta	1.0
Factor loadings	None
Estimated excess return volatility	13.2%

Note: Estimated excess return volatility is the assumed private equity manager idiosyncratic risk.
Source: Vanguard.

25 For further details on VCMM, see Davis et al. (2014).

IMPORTANT: The projections and other information generated by the VCMM regarding the likelihood of various investment outcomes are hypothetical in nature, do not reflect actual investment results, and are not guarantees of future results. Distribution of return outcomes from VCMM are derived from 10,000 simulations for each modeled asset class. Simulations as of December 31, 2019. Results from the model may vary with each use and over time. For more information, please see Appendix G.

VAAM is a proprietary model that solves these challenges by determining asset allocation among passive, active, and factor investments simultaneously. VAAM makes allocation decisions among assets under return uncertainty by adopting a utility-based optimization framework that seeks to maximize the expected utility of wealth. One of the main benefits of a utility-based framework is that it lends itself to consider investors' risk preferences toward uncertain portfolio outcomes. We have extended VAAM to two additional risk dimensions—active and factor risk. Therefore, VAAM optimized portfolios are consistent with investors' risk preferences across the dimensions of active-passive-factors.[26]

Incorporating private equity in VAAM

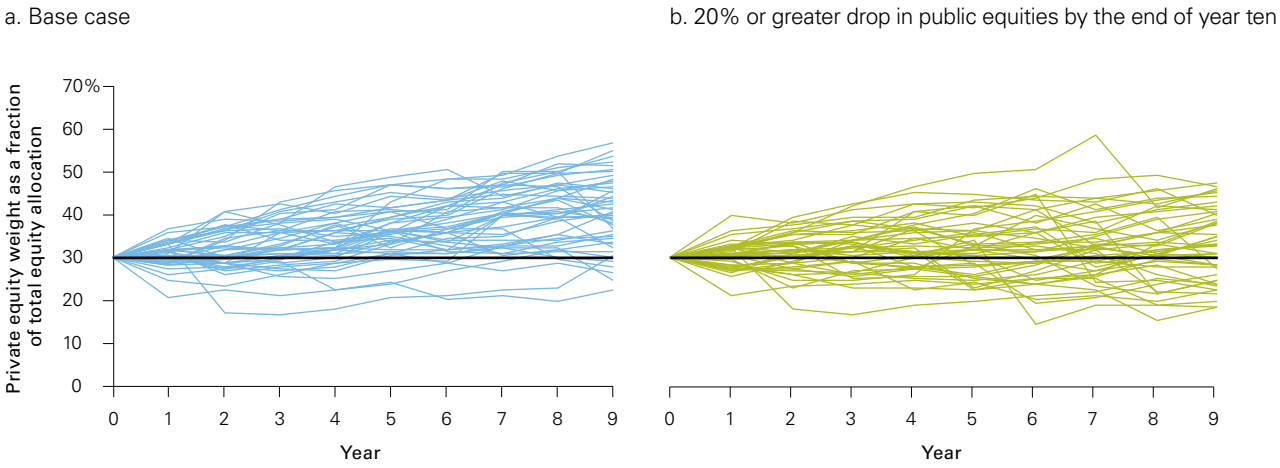
Important differences between private equity funds and active public market investments create challenges for incorporating private investments into VAAM. Accounting for and modeling these differences is critical for a sensible portfolio construction framework that can be extended to illiquid investments. Three main adjustments to the standard VAAM framework are needed:

1. *Model illiquidity-constrained portfolio rebalancing (drop the frictionless rebalancing assumption)*

The standard assumption in VAAM (as well as in most other portfolio construction frameworks) of frictionless and periodic rebalancing back to target asset allocation is invalid for illiquid assets such as private equity.[27] As discussed in Ma and Pirone (2014) and Baxter (2018), taking into account the inability to perfectly rebalance is critical when determining optimal asset allocation. Ang, Papanikolaou, and Westerfield (2014) propose an approach that accounts for the drift in the illiquid asset allocation (akin to a buy-and-hold strategy) when full frictionless rebalancing is not possible and the illiquid asset cannot be traded for an uncertain period of time.

Following these studies, we model an illiquidity constraint that prevents full rebalancing of private equity in VAAM. After the initial allocation is decided, the illiquid position is allowed to drift until the end of the investment horizon.[28]

Figure 8. Evolution of private equity weight over time under different hypothetical economic scenarios



Notes: Portfolios have been optimized over a ten-year investment horizon assuming a medium level of active risk aversion. Passive risk aversion in VAAM was set to a level consistent with a 70% stock/30% bond policy portfolio. The following constraints apply: non-U.S. bonds, up to 50% of total (noncredit) bonds; total credit bonds, up to 50% of total fixed income (bonds and credit bonds); intermediate-term U.S. credit bonds, up to 60% of total credit bonds; short-term U.S. credit bonds, up to 60% of total credit bonds. For these case studies, we assume that the only sub-asset class in the portfolio with active investments is private equity. The analysis for Figure 8a includes the entire set of 10,000 VCMM simulations, whereas Figure 8b includes only those simulation scenarios that are consistent with a minimum 20% drop in global public equity by the end of the investment horizon (a total of 3,439 simulations). In order to show the same number of simulated paths for both Figure 8a and Figure 8b, we illustrate only simulation paths of private equity weights at discrete percentiles of the entire distribution of private equity weights at the end of the horizon. We display percentiles ranging from 5% to 95% with a 2% step (i.e., 5th percentile, 7th percentile, 9th percentile . . . up to 95th percentile).

Source: Vanguard calculations, using asset-return projections from the VCMM as of December 31, 2019, in USD.

26 See Aliaga-Díaz et al., 2019, for a full description of VAAM.

27 Some rebalancing is possible via cash flows; for instance, attempting to match capital calls with distributions across multiple vintages. However, because cash flows are entirely out of the investor's control, achieving full rebalancing back to target is extremely unlikely. See the next subsection for a full discussion on this.

28 The two assumptions—frictionless rebalance versus illiquidity-constraint rebalance—can lead to significantly different portfolio weights, as the mathematics that represent each approach are different (see Appendix D).

This illiquidity-constrained rebalance feature yields a number of important benefits. Most important, it provides a more accurate representation of the risk-return trade-offs assessed within the portfolio optimization, particularly the trade-off between liquidity premium and liquidity risks associated with private equity. It also allows for the derivation of key portfolio analytics to assess illiquidity risks in the portfolio, such as the probability of breaching an investor's maximum private equity threshold at any point in time.

Figure 8 plots the evolution over the investment horizon of the rebalancing-constrained private equity allocation under two hypothetical economic scenarios based on VCMM projections: base case and a drop in equity of at least 20% by the end of the investment horizon. The charts show that the drifted weight for private equity can go above or below the initial weight. By knowing the number of simulations that lead the private equity weight to be above or below a certain threshold (e.g., 40%), we can compute the probability for the private equity weight to breach the threshold. This metric could be an important consideration for investors when finalizing their allocation to private equity. A high probability of breaching a specific private equity threshold may warrant constraining the initial allocation to private equity.

2. *Explicit modeling of private equity cash flows*

Also discussed previously is the issue of funding liquidity risk, stemming from the inflexibility and uncertainty in the timing and magnitude of the cash flows associated with a private equity position. Investors are exposed to funding liquidity risk because distributions and capital calls are decided solely by the private equity fund manager. On the distributions side, the fund's decision to pay out cash at unknown times and in uncertain magnitudes would tend to push the portfolio toward being underallocated over time. On the capital calls side, the inflexible and uncertain schedule of the drawdowns means that the risk of being overallocated at any point in time is significant.

Although commitments to new vintages or matching capital calls with distributions from previous vintages may provide some hedge to this risk, they will not eliminate the risk of significant overallocation or underallocation at any point in time. Also, equitized capital commitments combined with an adverse market environment mean that capital calls can lead to significant allocation shifts in other parts of the liquid portfolio.

Explicitly modeling cash flows in a multi-asset portfolio with private equity is therefore important, along with illiquidity considerations and relevant rebalancing rules (Siegel, 2008; Baxter, 2018). Two aspects are important to keep in mind when modeling cash flows:

- **Ex ante uncertainty about cash flows.** Although it is common to assume a deterministic stream of cash flows (both distributions and capital calls) for a given vintage in a given fund, this assumption may be less realistic for a laddered private equity position spanning multiple vintages. Assuming a distribution of cash flows (similar to the Monte Carlo simulation of asset returns) may render a more accurate description of this additional source of investment uncertainty in the portfolio.
- **Set explicit rebalancing rules for the uncertain cash flows.** Assumptions about what is done with cash flows are also an important modeling attribute. For instance, if net cash flow at a point in time is positive (distribution is larger than capital calls) and private equity is underallocated (below target), then positive cash flows may not help restore the target allocation in the short run, since committing to new private equity vintages may not trigger capital calls until a future uncertain date. Similarly, if net cash flow is negative and private equity is overallocated, then allocation will continue to drift above target. Figure 9 summarizes the possible rebalancing rules for all four scenarios.

Figure 9. Private equity cash flows and rebalancing rules

Current private equity allocation relative to target

Net cash flows (distributions minus capital calls)	Overallocated	Underallocated
Positive (distributions > capital calls)	Reinvest net cash flows in the liquid portfolio. Private equity allocation drifts back toward target.	Commit to new vintages. This may not correct the underallocation in the short term.
Negative (distributions < capital calls)	Fund net cash flows proportionally from the liquid portfolio. Private equity allocation remains overallocated.	Fund net cash flows proportionally from the liquid portfolio. Private equity allocation drifts back up toward target.

Source: Vanguard.

The objective of the rules described above is not to take on the daily rebalancing operation based on ex-post cash flows and performance of private equity but rather to capture, at a high level, the additional source of ex-ante uncertainty in the portfolio stemming from uncertain cash flow streams interacting with stochastic private equity valuations.

3. *Optional valuation adjustment of the illiquid wealth in the portfolio*

As discussed previously, liquidity risks mean that the capital invested in private equity is inaccessible to the investor for an extended period of time, unless through the secondary market, but at a discount to NAV in most cases (Siegel, 2008). This not the case with public liquid assets, where wealth can be accessed or "liquidated" freely. Thus, depending on the goal of each investor, it might be necessary to account for the disparity between the market value of liquid and illiquid assets in the portfolio at the end of the investment horizon.

Therefore, we add this valuation adjustment feature in VAAM. Similarly to Kinlaw, Kritzman, and Turkington (2013) and Hayes, Primbs, and Chiquoine (2015), we account for this liquidity disparity between private and public assets by specifying a penalty, or discounting factor, applied to private equity wealth in the portfolio. In other words, we are assuming that once the portfolio has to be valuated at the end of the investment horizon (e.g., ten years), the private equity portion of the portfolio will be assessed at market prices (e.g., in the secondary market), usually at a discount to NAV. The discount function effectively converts illiquid wealth into its liquid equivalent, such that private wealth can now be fairly traded off and compared with its public liquid counterparts. Based on Nadauld et al. (2019), we have assumed a 15% secondary market discount in the examples illustrated in the following sections.[29]

Multi-asset portfolios with private equity

How much capital should investors allocate between public and private illiquid assets? How sensitive is the allocation to investors' risk preferences and to different risk and return assumptions for private equity? In this section, we explore these questions using a new version of VAAM developed specifically to accommodate illiquid assets in long-term, strategic portfolios. The new features (discussed above), along with the private equity risk and return expectations from Figure 7, allow us to analyze several practical examples of private equity allocations, illustrating their key drivers and portfolio sensitivities. We analyze:

- The illiquidity-constrained rebalancing feature (versus an unrealistic frictionless rebalancing assumption).
- Varying degrees of investors' active risk aversion.
- Different levels of private equity manager excess return targets.
- Different levels of private equity manager idiosyncratic risk expectations.

The asset universe employed in the portfolios consists of a global equities index, short-term and intermediate-term U.S. credit bond indexes, a U.S. aggregate bond index, and a global-ex U.S. aggregate bond index (hedged to USD), in addition to a hypothetical private equity fund described in Figure 7.[30] The investment horizon is ten years. For simplicity of exposition, we ignore private equity cash flows and we assume that the investor will continue to roll over capital from private equity toward new vintages over the investment horizon.[31] Also, we are assuming that investors are using private equity funds rather than investing directly in private companies. Unless otherwise noted, portfolios have been optimized around a 70/30 passive, fully liquid policy portfolio (70% global equities and 30% global bonds).

The portfolios with private equity (and various sub-asset classes) are derived from VAAM for three levels of investor active risk aversion. **Figure 10** displays two sets of portfolios. Figure 10a shows VAAM portfolios including private equity allocations but ignoring entirely private equity's illiquid characteristics; namely, there is no valuation adjustment for illiquidity, and there is frictionless rebalancing to target. In contrast, Figure 10b shows portfolio allocations with an illiquidity-constrained rebalancing feature and illiquidity valuation discount for private equity. The optimized private equity weight here represents the optimal initial allocation to private equity, assuming that an investor starts with the full private equity allocation at time zero. As expected, accounting for illiquidity risks results in a lower allocation to private equity. More specifically, the private equity share within equities decreases substantially, even in the case of low-level active risk aversion, where it drops from 76% to 46%.[32]

29 The 15% discount used in the example is an average over the sample period used in Nadauld et al. (2019), which estimates a median purchase price as percentage to NAV of 85.6% using data pre-2006 to 2014. However, the appropriate discount used in actual applications of VAAM may be different and may need to be determined on a case-by-case basis.

30 Refer to Appendix E for details of the indexes used.

31 This implicitly assumes that private equity fund contributions and distributions offset each other at each period.

32 If the overall portfolio weight would be small, the investor has to determine if the expected incremental benefit is enough to justify portfolio inclusion given the practical complexities of private equity investing described in this paper.

Figure 10. VAAM-derived private equity allocations for a 70/30 policy portfolio

a. With a periodic rebalancing assumption

Active risk aversion	Portfolio		Metrics	
Lower	76%	Private equity share of total equity	Total equity	71%
	24%	Public equity share of total equity	PE share of total equity	76
	54%	Private equity	PE share of portfolio	54
	17%	Public equity	Expected return	8.0
	29%	Fixed income	Expected volatility	14.1
Medium	50%	Private equity share of total equity	Total equity	72%
	50%	Public equity share of total equity	PE share of total equity	50
	36%	Private equity	PE share of portfolio	36
	36%	Public equity	Expected return	7.1
	28%	Fixed income	Expected volatility	12.9
Very high	8%	Private equity share of total equity	Total equity	72%
	92%	Public equity share of total equity	PE share of total equity	8
	6%	Private equity	PE share of portfolio	6
	66%	Public equity	Expected return	5.5
	28%	Fixed income	Expected volatility	11.6

b. With an illiquidity-constrained rebalancing assumption

Active risk aversion	Portfolio		Metrics	
Lower	46%	Private equity share of total equity	Total equity	79%
	54%	Public equity share of total equity	PE share of total equity	46
	36%	Private equity	PE share of portfolio	36
	43%	Public equity	Expected return	7.3
	21%	Fixed income	Expected volatility	14.4
Medium	30%	Private equity share of total equity	Total equity	77%
	70%	Public equity share of total equity	PE share of total equity	30
	23%	Private equity	PE share of portfolio	23
	54%	Public equity	Expected return	6.8
	23%	Fixed income	Expected volatility	13.3
Very high	4%	Private equity share of total equity	Total equity	72%
	96%	Public equity share of total equity	PE share of total equity	4
	3%	Private equity	PE share of portfolio	3
	69%	Public equity	Expected return	5.4
	28%	Fixed income	Expected volatility	11.6

Notes: *PE* refers to private equity. Expected return and expected volatility are median values from a distribution of 10,000 simulations. Portfolios have been optimized over a ten-year investment horizon. Passive risk aversion in VAAM was set to a level consistent with a 70/30 policy, fully liquid portfolio. The following constraints apply: non-U.S. bonds, up to 50% of total (noncredit) bonds; total credit bonds, up to 50% of total fixed income (bonds and credit bonds); intermediate-term U.S. credit bonds, up to 60% of total credit bonds; short-term U.S. credit bonds, up to 60% of total credit bonds. For these case studies, we assume that the only sub-asset class in the portfolio with active investments is private equity. For the portfolios reported in Figure 10b, a 15% NAV discount rate was assumed on the private equity wealth at the end of the ten-year investment horizon. The inner ring shows the total portfolio allocation to each asset category. The outer ring shows the portion allocated to private equity out of the total allocation to global equities. The optimized weights for intermediate-term U.S. credit bonds, short-term U.S. credit bonds, and international bonds (hedged) have been aggregated in the "fixed income" class.

Source: Vanguard calculations, using asset-return projections from the VCMM as of December 31, 2019, in USD.

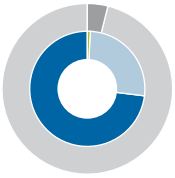
Figure 10b also illustrates the effect of varying levels of active risk aversion for the illiquid assumption. Allocations to private equity change significantly across risk aversion levels, ranging from 36% to 3% of the portfolio for low to very high aversion. Appendix F reports a summary of the 70/30 policy portfolios with private equity assuming perfect rebalance and a rebalancing constraint due to illiquidity.

Figure 11 shows private equity allocations for more conservative investors targeting a 30/70 policy portfolio assuming an illiquidity rebalancing constraint for private equity. Although the total equity allocation decreases significantly in line with the more conservative policy, the target private equity share of total equity does not change materially relative to that of the 70/30 investor in Figure 10b. In other words, for higher passive risk aversion levels, the weight of private equity decreases proportionally compared to the weight of total equity allocation. As an example, the private equity share of total equity only marginally decreases, from 30% (Figure 10b) to 28% (Figure 11) for an investor with a medium-level active risk aversion.

In general, a tradeoff between "alpha" allocations and "beta" exposures is typical of VAAM active-passive portfolios. When a high-information-ratio active asset (here given by the private equity performance assumptions) is introduced as an investment option, both the active-passive mix and the overall stock-bond split are impacted. Effectively, VAAM "makes room" in the overall portfolio risk budget for more of the active exposure by lowering the overall public equity allocation. This is evidenced by VAAM recommending an equity overweight relative to the policy in Figure 10, while it recommends overweighting and underweighting equities in Figure 11.

Figure 12 shows the magnitude of the private equity weight impact assuming a lower excess return expectation, from 4% per annum to 2% per annum (Figure 12a), and assuming a lower manager idiosyncratic risk (Figure 12b). For instance, for an investor with a medium active risk aversion, the private equity share of total equity almost halves from 30% (Figure 10b) to 16% (Figure 12a). Consistently, the expected return of the portfolios is reduced along with the standard deviation.

Figure 11. VAAM-derived private equity allocations for a 30/70 policy portfolio

Active risk aversion	Portfolio		Metrics	
Lower	45% Private equity share of total equity 55% Public equity share of total equity 15% Private equity 19% Public equity 66% Fixed income		Total equity PE share of total equity PE share of portfolio Expected return Expected volatility	34% 45 15 4.9 7.1
Medium	28% Private equity share of total equity 72% Public equity share of total equity 9% Private equity 24% Public equity 67% Fixed income		Total equity PE share of total equity PE share of portfolio Expected return Expected volatility	33% 28 9 4.5 6.4
Very high	4% Private equity share of total equity 96% Public equity share of total equity 1% Private equity 26% Public equity 73% Fixed income		Total equity PE share of total equity PE share of portfolio Expected return Expected volatility	27% 4 1 3.9 5.0

Notes: *PE* refers to private equity. Expected return and expected volatility are median values from a distribution of 10,000 simulations. Portfolios have been optimized over a ten-year investment horizon. Passive risk aversion in VAAM was set to a level consistent with a 30/70 policy portfolio. The following constraints apply: non-U.S. bonds, up to 50% of total (noncredit) bonds; total credit bonds, up to 50% of total fixed income (bonds and credit bonds); intermediate-term U.S. credit bonds, up to 60% of total credit bonds; short-term U.S. credit bonds, up to 60% of total credit bonds. For these case studies, we assume that the only sub-asset class in the portfolio with active investments is private equity. A 15% NAV discount rate was assumed on the private equity wealth at the end of the ten-year investment horizon. The inner ring shows the total portfolio allocation to each asset category. The outer ring shows the portion allocated to private equity out of the total allocation to global equities. The optimized weights for intermediate-term U.S. credit bonds, short-term U.S. credit bonds, and international bonds (hedged) have been aggregated in the "fixed income" class.

Sources: Vanguard calculations, using asset-return projections from the VCMM as of December 30, 2019, in USD.

Figure 12. Sensitivity of private equity allocations to different risk and return attributes

a. Lowering expected excess return from 4% to 2%

Active risk aversion	Portfolio		Metrics	
Lower	24%	Private equity share of total equity	Total equity	74%
	76%	Public equity share of total equity	PE share of total equity	24
	18%	Private equity	PE share of portfolio	18
	56%	Public equity	Expected return	5.8
	26%	Fixed income	Expected volatility	12.5
Medium	16%	Private equity share of total equity	Total equity	74%
	84%	Public equity share of total equity	PE share of total equity	16
	12%	Private equity	PE share of portfolio	12
	62%	Public equity	Expected return	5.7
	26%	Fixed income	Expected volatility	12.1
Very high	3%	Private equity share of total equity	Total equity	72%
	97%	Public equity share of total equity	PE share of total equity	3
	2%	Private equity	PE share of portfolio	2
	70%	Public equity	Expected return	5.2
	28%	Fixed income	Expected volatility	11.5

b. Lowering manager idiosyncratic risk from 13.2% to 9.2%

Active risk aversion	Portfolio		Metrics	
Lower	68%	Private equity share of total equity	Total equity	84%
	32%	Public equity share of total equity	PE share of total equity	68
	57%	Private equity	PE share of portfolio	57
	27%	Public equity	Expected return	7.9
	16%	Fixed income	Expected volatility	15.3
Medium	46%	Private equity share of total equity	Total equity	79%
	54%	Public equity share of total equity	PE share of total equity	46
	36%	Private equity	PE share of portfolio	36
	43%	Public equity	Expected return	7.2
	21%	Fixed income	Expected volatility	13.8
Very high	7%	Private equity share of total equity	Total equity	72%
	93%	Public equity share of total equity	PE share of total equity	7
	5%	Private equity	PE share of portfolio	5
	67%	Public equity	Expected return	5.4
	29%	Fixed income	Expected volatility	11.5

Notes: *PE* refers to private equity. Expected return and expected volatility are median values from a distribution of 10,000 simulations. Portfolios have been optimized over a ten-year investment horizon. Passive and active risk aversions are set to be identical to those in Figure 10, with only excess return (Figure 12a) and manager idiosyncratic risk (Figure 12b) changing. The following constraints apply: non-U.S. bonds, up to 50% of total (noncredit) bonds; total credit bonds, up to 50% of total fixed income (bonds and credit bonds); intermediate-term U.S. credit bonds, up to 60% of total credit bonds; short-term U.S. credit bonds, up to 60% of total credit bonds. For these case studies, we assume that the only sub-asset class in the portfolio with active investments is private equity. A 15% NAV discount rate was assumed on the private equity wealth at the end of the ten-year investment horizon. The inner ring shows the total portfolio allocation to each asset category. The outer ring shows the portion allocated to private equity out of the total allocation to global equities. The optimized weights for intermediate-term U.S. credit bonds, short-term U.S. credit bonds, and international bonds (hedged) have been aggregated in the "fixed income" class. Portfolio totals may not equal 100% due to rounding.

Sources: Vanguard calculations, using asset-return projections from the VCMM as of December 30, 2019, in USD.

On the other hand, private equity allocations seem to be even more responsive to a 4 percentage point drop in the manager idiosyncratic risk, from 13.2% to 9.2% per annum, where private equity weights increase across the board. For instance, for an investor with a medium active risk aversion, the private equity share of total equity goes from 30% (Figure 10b) to 46% (Figure 12b). Thus, the allocation to private equity is dependent on the excess return and idiosyncratic risk expectation and an investor's risk preferences across active and passive risk dimensions. VAAM helps explicitly derive asset allocations conditional on these inputs, while incorporating key characteristics of an illiquid asset such as illiquidity-constrained rebalancing, cash flows, and valuation adjustments for illiquidity.

Conclusion

Private equity is a growing and important component of the investing landscape. It provides investors the opportunity to gain exposure to companies that are not accessible in public markets and the potential to generate significant excess returns compared with public investments.

While private equity and public equity share a number of sources of risk and return, there are structural differences that create a unique set of risks and challenges for investors considering private equity for a diversified portfolio. The main differences are constraints on the ability to rebalance, inflexible and uncertain timing and size of cash inflows and outflows when owned through a fund structure, and a valuation adjustment for the illiquid assets in the portfolio. We use VAAM to construct optimized multi-asset portfolios that include private equity using different assumptions for investor risk tolerance and return and risk expectations for the sub-asset class. Our results indicate that assuming frictionless and regular rebalancing to and from private equity leads to overestimates in the optimal allocation, compared with a more realistic rebalancing under private equity liquidity constraints that investors face. We find that assuming higher levels of investor active risk aversion leads to a lower allocation to private equity, while lower expectations for active risk or higher expectations for alpha lead to larger allocations.

Although there is no single recommended allocation for all portfolios, private equity can play an important role in a long-term, diversified portfolio for disciplined investors who are able to identify and access high-quality managers or attractive direct investments and can accept the inherent risks.

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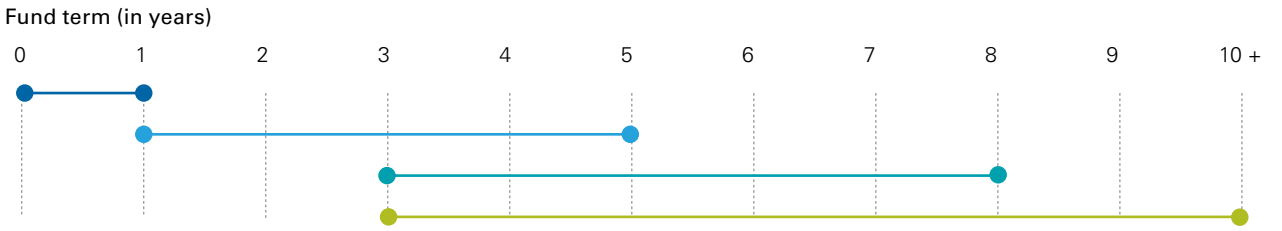
Appendix

Appendix A: Life cycle and styles of private equity funds

When a fund is first legally established, the manager begins attempting to gather capital commitments from investors, commonly referred to as the *fundraising period*. This period can end once the maximum desired dollar commitment level is reached or on a certain prespecified date. Investments can be made in portfolio companies during the first few years of the fund's life, referred to as the *investment period*, which is typically set to contractually end after about five or six years. After that, the manager cannot purchase additional companies but is sometimes allowed to make follow-on investments in existing portfolio companies. As investments are made, the managers typically supply expertise and networks to assist with value creation by portfolio companies and can negotiate the eventual sale (i.e., realization) of those investments before the end of the fund's contractual term. The sale can be to a strategic buyer (e.g., merger or acquisition), another private equity fund, a direct investor, or through an IPO. While the average life of a fund is about ten years, this term often can be extended a few years upon mutual agreement between the manager and the investors, if more time to exit portfolio holdings is deemed financially advantageous for investors. **Figure A-1** illustrates the typical life cycle of a private equity fund.

Many funds focus their investments in companies at certain stages of their growth cycle. For instance, *venture capital funds* provide equity capital to start-up or early stage privately owned companies with significant growth potential. The stakes typically make them minority shareholders. *Growth funds* are a category that sits between venture capital and buyout funds. Managers take a minority stake (typically with little or no debt) in profitable, maturing companies with opportunities for growth where the supplied capital is typically used to help finance specific expansion initiatives. In contrast, *buyout funds* typically acquire a controlling stake in mature companies that may be private or public at the time of purchase. A significant portion of the purchase price often consists of debt, and these funds essentially buy out existing owners, which is why they are often referred to as "leveraged buyouts." **Figure A-2** shows where along the company life cycle the desired opportunities are for different types of funds.

Figure A-1. Example of a private equity fund life cycle

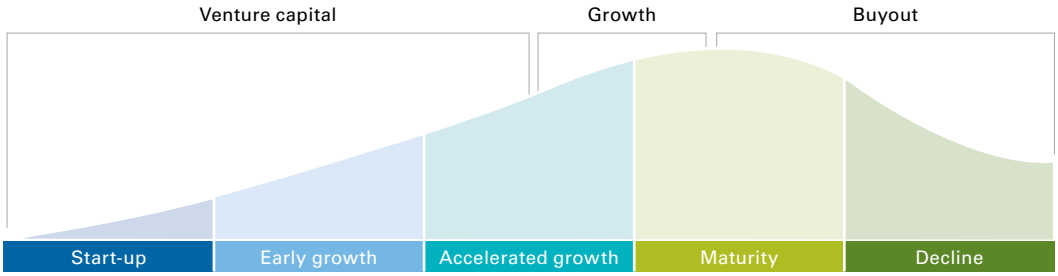


- **Fundraising period:** Manager gathers investor commitments
- **Investment period:** Manager "calls" money from investors as needed to fund investments
- **Value-enhancement period:** Manager helps improve portfolio companies
- **Divestment period:** Manager sells portfolio company holdings when appropriate and distributes net proceeds to investors

Notes: The timeline is hypothetical and for illustrative purposes only. These periods can vary by fund.
Source: Vanguard.

Figure A-2. Private equity funds often target companies in specific growth stages

Desired opportunities for different types of funds along the development of portfolio companies



Source: Vanguard.

Appendix B: Deciding whether to custom-build a portfolio of funds or outsource to a fund of funds

Harris et al. (2018) finds that the performance of private equity funds of funds compared with that of various hypothetical portfolios of funds is mixed. Given this empirical evidence and the various trade-offs involving ease of access to managers, costs, and ability to evaluate managers, the prudence of using a fund of funds should be assessed on a case-by-case basis. Figure B-1 summarizes this assessment.

Figure B-1. Building a private equity allocation with funds or funds of funds: Key considerations

	"Do-it-yourself" mix of funds	Funds of funds
Diversification	Requires a large capital commitment to build a diversified portfolio of funds.[33]	Pooled capital from multiple investors achieves scale to diversify exposure across geographies, stages, industries, and managers.
Selection and monitoring skill	Investors need expertise to conduct high-quality due diligence and properly build and manage a private equity portfolio.	Investors should assess whether their expertise is better than that of a third party. Experienced fund-of-funds managers may participate on underlying private equity fund advisory committees. On those committees, they can monitor the manager more closely, oversee fund valuations, and participate in decisions on potential conflict-of-interest transactions and waivers of certain restrictions or thresholds set forth in the fund's legal documents (Scharfman, 2018).
Access	Requires the prestige and capital necessary to access high-quality managers.	Some funds of funds may have more access to higher-quality, oversubscribed funds with proven managers and direct investment opportunities as a result of their early investment with that manager or their reputation as a valuable limited partner.
Cost	Expected costs include internal staff and any consultants who help build a private equity portfolio in-house and manage the cash flows and contracts.	Separate management and performance fees are typically charged, on top of what is paid to the underlying fund managers.[34] The organizational scale of the fund-of-funds manager may help during fee negotiations with the underlying fund managers.

Sources: Vanguard and Harris et al. (2018)

33 Using data from Preqin from 2015 to 2020, the median minimum investment requirement for venture capital and buyout funds, as well as funds of funds, was $1 million. It is important to note that some high-quality managers who are accessible have minimum investment requirements significantly higher than the median, particularly for buyout funds. As a simple hypothetical example, consider that an investor has a 20% target portfolio allocation to private equity and wants to commit to 20 high-quality private equity funds that each have a minimum requirement of $10 million. The investor would have to commit at least ($10,000,000 x 20 funds) = $200 million, and the investor's total portfolio would have to be at least ($200,000,000 / 20%) = $1 billion.

34 According to Døskeland and Strömberg (2018), the median fund-of-funds management fee was 1.0%, with a median carried interest fee of 10%. Naturally, the investor would incur some additional costs to conduct a search for and monitor any hired fund-of-funds manager(s).

Appendix C. Equity style factor definitions

Figure C-1 shows the criteria used to define and construct the equity style factors for the analysis reported in this paper.

Appendix D. The mathematics behind the illiquidity of portfolio rebalancing versus rebalanced strategies

For liquid assets, which are rebalanced back to the optimal asset allocation on a regular basis over the investment period, we compute the level of portfolio wealth for a rebalanced strategy at maturity as:

$$W_T^{Reb} = \prod_t \left(\sum_i x_i (1 + r_{i,t}) \right) = \prod_t \left(\sum_i x_i + \sum_i x_i r_{i,t} \right) =$$

$$\prod_t \left(1 + \sum_i x_i r_{i,t} \right)$$

where x_i is the weight for asset class i, and $r_{i,t}$ is its return at period t. Having the optimal weights x_i "inside" the product operator makes the weight in the liquid investment be computed as if it were resetting to its optimal target at each point in time.

A strategy assuming imperfect rebalancing instead is one where the initial allocation is decided and the investment is then held without making any changes until maturity. The wealth at maturity for a rebalancing constraint strategy can be expressed as:

$$W_T^{BaH} = \sum_i \left(x_i \prod_t (1 + r_{i,t}) \right)$$

Note how in this case the optimal weight x_i is "outside" the product operator. The constrained strategy assumes the optimal weights to be fixed at $t=0$ such that it is optimal to let the investment drift over time, with no need to readjust the asset allocation.

Appendix E. Indexes assumed in the portfolio analysis

- **Global equities:** MSCI All Country World Index
- **Short-term U.S. credit bonds:** Bloomberg Barclays U.S. 1–3 Year Credit Bond Index
- **Intermediate-term U.S. credit bonds:** Bloomberg Barclays U.S. 3–7 Year Credit Bond Index
- **U.S. aggregate bonds:** Bloomberg Barclays U.S. Aggregate Bond Index
- **Global-ex U.S. aggregate bonds (hedged to USD):** Bloomberg Barclays Global Aggregate ex-USD Index

Figure C-1.

Factor	Succinct definition	Selection universe	Weighting scheme
Global market	Broad global coverage across developed and emerging markets	MSCI All Country World Index	Market-capitalization-weighted
Global value	1/3 of stocks with the lowest price-to-book ratio	MSCI All Country World Index	Market-capitalization-weighted
U.S. small-cap	2/3 of stocks with lowest market capitalization	Russell 3000 Index	Market-capitalization-weighted

Appendix F. Summary of VAAM-derived private equity allocations

Lower active risk aversion		70/30 policy			30/70 policy
	Periodic rebalancing			Illiquidity-constrained rebalancing	Illiquidity-constrained rebalancing
	Base	Base	Lower excess return	Lower manager idiosyncratic risk	Base
PE share of total equity	76%	46%	24%	68%	45%
PE share of portfolio	54%	36%	18%	57%	15%
Total equity	71%	79%	74%	84%	34%
Expected return	8.0%	7.3%	5.8%	7.9%	4.9%
Expected volatility	14.1%	14.4%	12.5%	15.3%	7.1%
Expected Sharpe ratio	0.40	0.33	0.27	0.36	0.36

Medium active risk aversion		70/30 policy			30/70 policy
	Periodic rebalancing			Illiquidity-constrained rebalancing	Illiquidity-constrained rebalancing
	Base	Base	Lower excess return	Lower manager idiosyncratic risk	Base
PE share of total equity	50%	30%	16%	46%	28%
PE share of portfolio	36%	23%	12%	36%	9%
Total equity	72%	77%	74%	79%	33%
Expected return	7.1%	6.8%	5.7%	7.2%	4.5%
Expected volatility	12.9%	13.3%	12.1%	13.8%	6.4%
Expected Sharpe ratio	0.36	0.32	0.27	0.34	0.35

Very high active risk aversion		70/30 policy			30/70 policy
	Periodic rebalancing			Illiquidity-constrained rebalancing	Illiquidity-constrained rebalancing
	Base	Base	Lower excess return	Lower manager idiosyncratic risk	Base
PE share of total equity	8%	4%	3%	7%	4%
PE share of portfolio	6%	3%	2%	5%	1%
Total equity	72%	72%	72%	72%	27%
Expected return	5.5%	5.4%	5.2%	5.4%	3.9%
Expected volatility	11.6%	11.6%	11.5%	11.5%	5.0%
Expected Sharpe ratio	0.26	0.26	0.25	0.26	0.32

Notes: Expected return, expected volatility, and expected Sharpe ratio are median values from a distribution of 10,000 simulations. Portfolios have been optimized over a ten-year investment horizon. The following constraints apply: non-U.S. bonds, up to 50% of total (noncredit) bonds; total credit bonds, up to 50% of total fixed income (bonds and credit bonds); intermediate-term U.S. credit bonds, up to 60% of total credit bonds; short-term U.S. credit bonds, up to 60% of total credit bonds. For these case studies, we assume that the only sub-asset class in the portfolio with active investments is private equity. For the illiquidity-constrained rebalancing portfolios, a 15% NAV discount rate was assumed on the private equity wealth at the end of the ten-year investment horizon. The base case portfolios refer to the standard equity/bond mix policy portfolios (Figure 10a, Figure 10b, and Figure 11), whereas the lower excess return and lower manager idiosyncratic risk portfolios refer to the sensitivity testing portfolios (Figure 12a and Figure 12b).

Sources: Vanguard calculations, using asset-return projections from the VCMM as of December 31, 2019, in USD.

Appendix G. About the Vanguard Capital Markets Model

IMPORTANT: The projections and other information generated by the Vanguard Capital Markets Model regarding the likelihood of various investment outcomes are hypothetical in nature, do not reflect actual investment results, and are not guarantees of future results. VCMM results will vary with each use and over time.

The VCMM projections are based on a statistical analysis of historical data. Future returns may behave differently from the historical patterns captured in the VCMM. More important, the VCMM may be underestimating extreme negative scenarios unobserved in the historical period on which the model estimation is based.

The VCMM is a proprietary financial simulation tool developed and maintained by Vanguard's Investment Strategy Group. The model forecasts distributions of future returns for a wide array of broad asset classes. Those asset classes include U.S. and international equity markets, several maturities of the U.S. Treasury and corporate fixed income markets, international fixed income markets, U.S. money markets, commodities, and certain alternative investment strategies. The theoretical and empirical foundation for the Vanguard Capital Markets Model is that the returns of various asset classes reflect the compensation investors require for bearing different types of systematic risk (beta). At the core of the model are estimates of the dynamic statistical relationship between risk factors and asset returns, obtained from statistical analysis based on available monthly financial and economic data. Using a system of estimated equations, the model then applies a Monte Carlo simulation method to project the estimated interrelationships among risk factors and asset classes as well as uncertainty and randomness over time. The model generates a large set of simulated outcomes for each asset class over several time horizons. Forecasts are obtained by computing measures of central tendency in these simulations. Results produced by the tool will vary with each use and over time.